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MYERS ARITHMETIC

BOOK II

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MYERS ARITHMETIC

BOOK II

BY

GEORGE WILLIAM MYERS

PROFESSOR OF THE TEACHING OF MATHEMATICS AND ASTRONOMY
COLLEGE OF EDUCATION, THE UNIVERSITY OF CHICAGO

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PREFACE

The work of Book I of the MYERS ARITHMETICS is for the third and fourth grades. It aims, first, to furnish clear concepts of number, and of the four fundamental arithmetical processes as tools for all sorts of measuring, and, second, to secure fairly good control of the necessary tabular machinery, including the facts of the addition and multiplication tables, the correlative subtraction and division truths, and the most useful facts and relations of denominate numbers. This is accomplished, partly by direct formal teaching of the tables, and partly by indirect uses in interesting problems, drawn from natural sources, suitably graded, and organized to impress the number lessons to be taught.

The broader educational aim of the work of Book II is to strengthen the grasp of arithmetic that is already secured, to give a measure of control of connected thinking, of inference-making, and to add appreciably to the pupil's ability to work independently. More particularly, this book seeks to expand the child's horizon of number to include fractional number, and to extend his control of the four fundamental operations to fractional number, both common and decimal. Long division, with its complicated technique, is thoroughly taught in Part I. Considerable systematizing of the pupil's knowledge of common fractions, together with a reconnoitre of decimals, and a forecast of percentage are also included in this Part.

Part II gives a topical first-treatment of addition, subtraction, multiplication, and division of whole and fractional number. It strengthens the hold on denominate numbers, on scale-drawing and mensuration, on bills and

accounts, and on percentage, and opens the attack upon interest. A little practical work on graphing is given. This constitutes Part II of the book, an arithmetic in itself. The first part of Book II is predominantly, not exclusively, spiral in treatment; the second part is predominantly, not exclusively, topical in its treatment.

The plan of these books recognizes the following important truths:

In the primary grades, children are too immature to profit by the study of topical unities, that call for much concentration. Unities, indeed, there must be; but they must be small enough to furnish immediate ends, and to lead to speedy successes. Arithmetical methods in the primary school must accordingly be characterized by variety, and by small topical unities. The spiral plan is well adapted to these grades.

In the grammar school, if work is to hold the interest, it must call for greater concentration. The pupil is now too mature either to enjoy or to profit by the practice of flitting from topic to topic at the beck of book or teacher. Unities of subject-matter must be larger and more complete than in the preceding grades. Pupils now work profitably and pleasurably under ends more remote, and successes may come a little more tardily. Gratuitous hopping from topic to topic, abiding nowhere long enough to make an appreciable point, is the sure road to failure in the grammar grades. The spiral plan is too lax, too centrifugal in its effects upon attention. The topical plan is best here. Grammar grade pupils crave the feeling that things are being completed as one subject is being left for another. Their standards of completeness may be unsophisticated. They are, however, none the less real. Under-concentration in the grammar school is as disastrous to interest as is over-concentration in the primary school.

In the intermediate grades, the pupil's mental attitude is transitional. His mode of working is something be-

tween the highly varied mode of the primary school and the more concentrative mode of the grammar school. The treatment best adapted to fifth and sixth grade pupils is a spiro-topical plan. Changes from topic to topic in these grades, particularly in the sixth, should be made at natural breaking points of subjects, or where minor or partial unities are completed. It is through the connectedness of these partial unities of topics that the child aids the natural drift of his mental faculties from the verbal to the rational type. This transition means much for arithmetic work, and good arithmetic work here will do much to promote the transition.

The destruction of interest in elementary school mathematics is nowhere so serious today as it is in the fifth and sixth grades. This is due, in good degree, at least, to the failure of arithmetical methodology to make practical recognition of the transitional type of mind found in these grades. Whether this book overcomes this difficulty or not, it, at least, assumes its reality and struggles with it.

Summaries of Parts I and II are given at the end of the book. These summaries serve (1) to assist the teacher to take an inventory of the results accomplished by the work of a year, and (2) to test the work with a view to guaranteeing a definite outcome. Furthermore, the teacher will welcome the summary of the work of a preceding grade as an aid in determining where and how to begin her work. The summaries will tend to reduce the waste due to "lost motion" from grade to grade in arithmetic.

The acknowledgments rendered in the preface of Book I deserve to be repeated here.

Mr. F. W. Buchholz's energy, vigilance, and patience with the manuscript and proofs have increased as the work on the series has advanced.

THE AUTHOR.

Chicago, June, 1908.

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PART I

1. How many rods is it from the middle of 12th St. to the middle of 13th St. (from B to C)? Scale, 1″:32 rd.

2. How many rods is it from C to D? From D to E? From E to F?

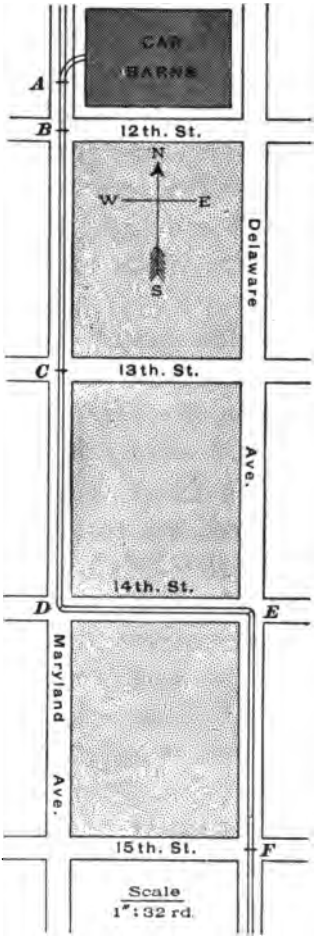
3. How many rods is it from A to B?

4. How many rods is it from A to D? From A to D to E to F?

5. The car-track runs on south 160 rods beyond F in Delaware Ave., and then turns east. How many rods is it from A to where the track turns east out of Delaware Ave.?

6. There are $5\frac{1}{2}$ yards in 1 rod. How many yards is it from A to B?

7. How many yards is it from B to C? From A to D to E? From A to D to E to F?



1. How many yards is it from F (page 1) to where the track turns east out of Delaware Ave.?

2. How many yards long is the track from A to where it leaves Delaware Ave.?

3. There are $16\frac{1}{2}$ feet in 1 rod. How many feet is it from A to B? From B to C? From C to D? From D to E? From E to F?

4. How many feet is it from F to where the track leaves Delaware Ave.?

5. How many feet long is the track from A to where it leaves Delaware Ave.?

6. The distance from A to where the track leaves Delaware Ave. is 1 mile. How many rods are there in 1 mile?

7. How many yards are there in 1 mile? How many feet are there in 1 mile?

8. In the drawing how wide are the streets and avenues? How wide are the actual streets and avenues?

9. In the drawing how far is it from the middle of 12th street due south to the middle of 13th street? How many rods is it between the middles of the actual streets?

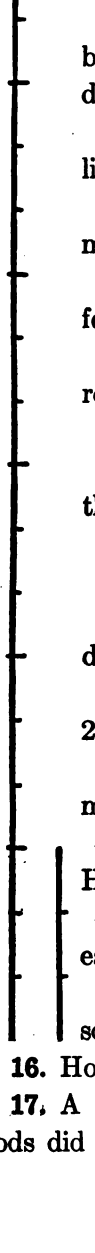
10. How many rods is it between the middle of Maryland avenue and the middle of Delaware Avenue?

11. How many north and south blocks, measured from middle to middle of streets, would make a mile?

12. How many east and west blocks, measured from middle to middle of avenues, would make a mile?

13. On the scale of the drawing (1 inch to 32 rods) how long a line would represent 1 mile?

14. How many eighths of an inch wide is the drawing of the car-barn? How many rods wide is the car-barn?



1. Measure these two lines. Suppose them to be drawn to the scale of 1 inch to 3 feet. What does the short line represent?

2. How many times the short line is the long line?

3. If the short line represents 1 yard, how many yards does the long line represent?

4. If the short line represents 3 feet, how many feet does the long line represent?

5. Call the distance the long line represents 1 rod. How many yards make a rod? How many feet?

6. How many feet make a yard?

7. How many inches make a foot? 320 times the distance the long line represents is a mile.

8. How many yards in a mile? Feet in a mile?

9. How many rods in a mile?

10. A boy walks 60 rods. How many yards does he walk? How many feet?

11. A lot 6 rods wide is divided lengthwise into 2 equal lots. How many feet wide is each lot?

12. A lot is 3 rods wide and 6 rods long. How many yards around it?

13. A bridge is 8 rods long and 2 rods wide. How many feet long is it? How many feet wide?

14. A rope 12 rods long is wound into coils, each coil using 6 feet. How many coils are there?

15. How many rods is it around a farm 2 miles square?

16. How many rods is it around a farm 3 miles square?

17. A bicyclist rode 30 miles in 1 hour. How many rods did he ride per minute? Per second?

1. How many yards is it from F (page 1) to where the track turns east out of Delaware Ave.?

2. How many yards long is the track from A to where it leaves Delaware Ave.?

3. There are $16\frac{1}{2}$ feet in 1 rod. How many feet is it from A to B? From B to C? From C to D? From D to E? From E to F?

4. How many feet is it from F to where the track leaves Delaware Ave.?

5. How many feet long is the track from A to where it leaves Delaware Ave.?

6. The distance from A to where the track leaves Delaware Ave. is 1 mile. How many rods are there in 1 mile?

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13. On the scale of the drawing (1 inch to 3? ro long a line would represent 1 mile?

14. How many eighths of an inch wide of the car-barn? How many rods wide

1. Measure these two lines. Suppose them to be drawn to the scale of 1 inch to 3 feet. What does the short line represent?

2. How many times the short line is the long line?

3. If the short line represents 1 yard, how many yards does the long line represent?

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5. Call the distance the long line represents 1 rod. How many yards make a rod? How many feet?

6. How many feet make a yard?

7. How many inches make a foot? 320 times the distance the long line represents is a mile.

8. How many yards in a mile? Feet in a mile?

9. How many rods in a mile?

10. A boy walks 60 rods. How many yards does he walk? How many feet?

11. A lot 6 rods wide is divided lengthwise into 2 equal lots. How many feet wide is each lot?

12. A lot is 3 rods wide and 6 rods long. How many yards around it?

13. A bridge is 8 rods long and 2 rods wide. How many feet long is it? How many feet wide?

14. A rope [illegible] around [illegible] miles [illegible] 2 miles

TABLE OF EQUIVALENTS

1 foot	1 yard	1 rod	1 mile
12 inches	3 feet	$5\frac{1}{2}$ yards	320 rods
	36 inches	$16\frac{1}{2}$ feet	1760 yards
			5280 feet

1. A block is 22 rods wide and 33 rods long. How many yards is it around the block? How many feet?

2. A boy lives 64 rods from school. If he makes 4 trips a day how many yards does he go in 5 days?

3. A man lives $2\frac{1}{2}$ miles from his office. He goes to his office each day. How many rods does he travel from Monday morning until Saturday night?

4. A lot is $82\frac{1}{2}$ feet wide and 165 feet long. What is the cost of fencing it at 18¢ a yard? At 75¢ a rod?

5. The rails on a railroad are 2 rods long. How many rails are there in a mile of single track?

6. It cost 85¢ a linear foot for making a street. What was the cost of a quarter of a mile of this street?

7. A boy rode $7\frac{1}{8}$ miles on his wheel. How many rods did he ride?

8. John's school is 38 yards west from his home. He walked to the store 45 yards east and then to school. How many yards did he walk?

9. The tire of a wheel measures 6 feet. How many revolutions will it make in going 24 rods?

10. A boy steps 2 feet. How many yards does he go in 33 steps? How many rods? How many steps will he take in walking $\frac{1}{2}$ of a mile? $\frac{1}{4}$ of a mile?

11. If there are 8 blocks in a mile, how many feet are there in a block? How many rods?

12. A field is $\frac{1}{2}$ mile square. How many miles does a man travel in going around the field?

1. Suppose 1 inch on this line stands for (represents) a mile. A man goes from C to A and back to B. How many miles has he traveled?

2. From B he goes to C and returns to B. How many miles has he traveled?

3. Suppose one inch on the line represents 8 rods. How many yards is it from A to B? From B to C? From A to C?

4. John lives at B, Ned at A, and the schoolhouse is at C. John goes to Ned's house in the morning, then to school, and home in the evening. How many rods has he traveled?

5. The next day Ned goes to school, back to John's home, and they go back to school together. They return home in the evening. How many rods has Ned traveled? How many yards?

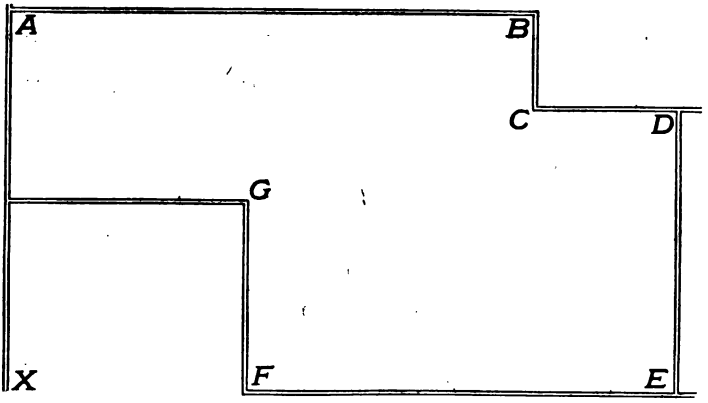
6. Let each pupil count the number of steps to his home from school, write it down, and the next day find the distance in feet, if his steps are 2 feet long. If he steps $1\frac{1}{2}$ feet. Tell whether it is nearer a mile, $\frac{1}{2}$ a mile, $\frac{1}{4}$ of a mile, or $\frac{1}{8}$ of a mile.

7. How many feet in $\frac{1}{2}$ of a mile? In $\frac{1}{8}$ of a mile?

8. 2 boys start from home and walk in opposite directions. They take 8 steps to the rod. How many rods apart are they after each has taken 80 steps?

9. Is their distance apart nearest to $\frac{1}{4}$, $\frac{1}{2}$, or 1 mile?

10. A boy takes 80 steps of 2 feet each in a minute. How far will he walk in 15 minutes? Is the distance nearer to $\frac{1}{4}$, or to $\frac{1}{2}$ of a mile?



Map of country road, starting from the street AX, drawn to a scale of one inch to four miles.

1. How many miles is it from the street to B? To D? To G?

2. How many miles is it from B to G? From D to G?

3. How far is the road B C from the street? How far is D E?

4. How far is the road F G from the road D E?

5. How far is the road F G from the street?

6. How far is the road A B from the road F E?

7. How far is G from the nearest point of the street?

8. How far is G from the nearest point in the road A B?

9. The distance, B C, equals what part of the distance, F G? What part of D E? What part of C D? What part of A B? What part of D F?

10. The distance, F G, equals what part of the distance A B? What part of the distance, D E?

11. The distance, C D, equals what part of the distance A B? What part of the distance, F G?

(On the map of the opposite page call 1 inch $\frac{1}{2}$ mile.)

1. How far is it from A to B? From A to C? From A to D? From B to C? From E to F? From E to G?

(On the map of the opposite page call 1 inch $\frac{1}{4}$ mile.)

2. How far is it from A to B? From A to C?

3. If a man walks from A to B in 15 minutes, how long will it take him to walk from D to E?

4. John rides his bicycle $1\frac{1}{2}$ miles in 8 minutes; Harry rides at the same speed for 5 minutes. How many more rods does John ride than Harry rides?

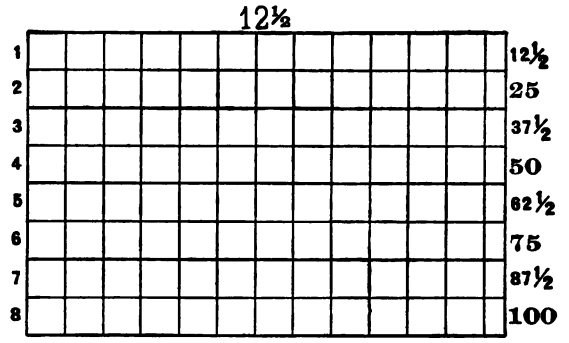
5. There are in a room 5 windows 9 feet high. How many yards of material of single width will be required to make 2 curtains for each of the windows? What will be the cost at 6¢ a yard?

6. The 4 windows of a schoolroom are 7 feet high. How many yards of material will be required for one curtain at each window, allowing 1 extra foot for each curtain? What will be the cost of the material at 25¢ a yard?

7. Each step in the staircase is 6 inches high and 1 foot wide. How many feet of stair carpet will be required for 12 steps? What will be the cost of the carpet at 75¢ a yard?

8. In a library $35\frac{1}{2}$ feet long and $23\frac{1}{2}$ feet wide, there are book shelves on one side and one end of the room. How many feet of boards, in length, will be required to make 5 of these shelves?

9. A pile of 10 blocks is placed 6 yards from a basket. A child starts at the basket, goes to the blocks and carries them 1 at a time to the basket. How far does he walk?



1. Add:

$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$
$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$
	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$
		$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$
			$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$
				$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$
					$12\frac{1}{2}$	$12\frac{1}{2}$
						$12\frac{1}{2}$

2. $12\frac{1}{2} \times 2 =$ .....? $12\frac{1}{2} \times 6 =$ .....?

$12\frac{1}{2} \times 3 =$ .....? $12\frac{1}{2} \times 7 =$ .....?

$12\frac{1}{2} \times 4 =$ .....? $12\frac{1}{2} \times 8 =$ .....?

$12\frac{1}{2} \times 5 =$ .....? $12\frac{1}{2} \times 10 =$ .....?

3. $25 \times 2 =$ .....? $25 - 12\frac{1}{2} =$ .....?

$25 \times 3 =$ .....? $50 - 12\frac{1}{2} =$ .....?

$25 \times 4 =$ .....? $75 - 12\frac{1}{2} =$ .....?

$50 \times 2 =$ .....? $100 - 12\frac{1}{2} =$ .....?

4. 25 is what part of 50? Of 75? Of 100? 50 is what part of 75? Of 100? 75 is what part of 100?

5. $12\frac{1}{2}$ is what part of 25? Of 50? Of 75? Of 100?

6. How many times $12\frac{1}{2}$ is $37\frac{1}{2}$? $62\frac{1}{2}$? $87\frac{1}{2}$?

7. What part of 100 is $37\frac{1}{2}$? $62\frac{1}{2}$? $87\frac{1}{2}$?

Answer all you can orally

1. A woman sold 4 dozen eggs at $12\frac{1}{2}¢$ a dozen. How much did she receive for them?

2. How many $12\frac{1}{2}¢$ make a dollar? $\frac{1}{4}$ of a dollar? A half-dollar? $75¢$?

3. A boy bought a dozen chickens at $12\frac{1}{2}¢$ each. The feed cost him $75¢$. He sold the chickens when they were grown at $25¢$ each. How much money did he make?

4. A farmer hired a boy to watch his corn-field and promised him $12\frac{1}{2}¢$ for every 3 squirrels and $12\frac{1}{2}¢$ for every 5 crows that he killed. At the end of a week the boy turned in 6 squirrels and 10 crows. How much should the farmer pay him?

5. A woman bought 10 yards of ribbon at $12\frac{1}{2}¢$ a yard and 8 yards of silk at $87\frac{1}{2}¢$ a yard. What did the ribbon cost? The silk? Both together?

6. A boy bought a ball for $12\frac{1}{2}¢$, a bat for $25¢$, and a glove for $37\frac{1}{2}¢$. How much did he pay for the outfit?

7. A girl bought a doll for $25¢$, a tablet and pencil for $12\frac{1}{2}¢$, and a book for $37\frac{1}{2}¢$. How much change should she receive if she gave the storekeeper $1?

8. A book-seller bought 5 books at $12\frac{1}{2}¢$ each, 8 boxes of paper at $12\frac{1}{2}¢$ each, and 7 dozen pencils at $12\frac{1}{2}¢$ a dozen. What was the cost of all?

9. A hall, $12\frac{1}{2}$ feet wide, is 6 times as long as wide. How long is it?

10. A board, $12\frac{1}{2}$ inches wide, is 7 times as long as wide. How many inches long is it?

11. A boy bought 3 dozen eggs for $\frac{3}{8}$ of a dollar, and sold them for $\frac{3}{4}$ of a dollar. What part of a dollar did he gain? How many cents a dozen did he gain?

1. A clerk sold 1 piece of silk cord $4\frac{1}{2}$ feet long, another $7\frac{1}{2}$ feet long. How many yards did he sell in all?

2. A man wishes to put 2 rows of wire above a fence 12 rods long. How many feet of wire does he need?

3. How many yards of carpet are needed to lay one width in a hall $22\frac{1}{2}$ feet long and on a flight of 16 stairs, each step requiring $1\frac{1}{2}$ feet of carpet?

4. John rides 12 miles, and his brother $\frac{3}{4}$ as far. How many rods does his brother ride?

5. A field is 160 rods long and 80 rods wide. How many feet of wire will go round it twice?

6. A man left home and drove 5 miles east; turned and drove 1000 rods back. How far was he from home?

7. The hall of a hotel is 14 yards long and $16\frac{1}{2}$ feet wide. How many feet of border will be required to go around the walls?

8. How many boards 12 feet long will make a fence 1 mile long, if there are 3 rows of boards?

9. One walk is 150 feet long, one is 80 feet, and one is 240 feet. How many yards long are all together?

10. How many miles long is a track having on one side 352 rails, each 30 feet long?

11. A street-car company lays 7 miles of track, $\frac{1}{4}$ of it running east and west, the rest north and south. How many rods of track are there in all? How many east and west? North and south?

12. A boy walked 2 miles, taking steps 2 feet long. How many steps did he take?

13. A farmer built a fence around a field $\frac{3}{4}$ of a mile long and $\frac{1}{2}$ of a mile wide. He used old material for 2250 yards. How many yards of new fencing did he use?

Answer all you can orally

1. Give the number of

3's in			11's in		12's in		
12	16	36	77	110	24	30	84
24	27	15	55	99	42	144	66
10	22	21	132	33	18	132	108
18	30	14	66	121	96	60	72
19	37	33	88	44	48	36	120

2.

$12 \times 4 =$.....?	$12 \times 5 =$.....?	$3 \times 11 =$.....?
$12 \times 9 =$.....?	$12 \times 3 =$.....?	$3 \times 12 =$.....?
$12 \times 7 =$.....?	$12 \times 12 =$.....?	$3 \times 5 =$.....?
$12 \times 10 =$.....?	$12 \times 6 =$.....?	$3 \times 8 =$.....?
$12 \times 8 =$.....?	$12 \times 11 =$.....?	$3 \times 9 =$.....?

3.

$12 \times 2\frac{1}{2} =$.....?	$12 \times 10\frac{1}{4} =$.....?	$3 \times 8\frac{1}{3} =$.....?
$12 \times 5\frac{1}{4} =$.....?	$12 \times 9\frac{1}{3} =$.....?	$3 \times 5\frac{1}{3} =$.....?
$12 \times 7\frac{1}{3} =$.....?	$12 \times 3\frac{1}{2} =$.....?	$3 \times 7\frac{1}{3} =$.....?
$12 \times 6\frac{1}{4} =$.....?	$12 \times 12\frac{1}{4} =$.....?	$3 \times 9\frac{1}{3} =$.....?
$12 \times 8\frac{1}{2} =$.....?	$12 \times 4\frac{3}{4} =$.....?	$3 \times 12\frac{1}{3} =$.....?

4. $5\frac{1}{2}$ is what part of 11? What part of $16\frac{1}{2}$? Of 22?

5.

$5\frac{1}{2} \times 2 =$.....?	$16\frac{1}{2} \times 3 =$.....?	$11 \times 8 =$.....?
$5\frac{1}{2} \times 4 =$.....?	$16\frac{1}{2} \times 6 =$.....?	$11 \times 9 =$.....?
$5\frac{1}{2} \times 7 =$.....?	$16\frac{1}{2} \times 4 =$.....?	$11 \times 12 =$.....?
$5\frac{1}{2} \times 8 =$.....?	$16\frac{1}{2} \times 2 =$.....?	$11 \times 4 =$.....?
$5\frac{1}{2} \times 6 =$.....?	$16\frac{1}{2} \times \frac{1}{3} =$.....?	$11 \times 7 =$.....?
$5\frac{1}{2} \times 9 =$.....?	$16\frac{1}{2} \times \frac{2}{3} =$.....?	$11 \times 10 =$.....?
$5\frac{1}{2} \times 3 =$.....?	$16\frac{1}{2} \times 1\frac{1}{3} =$.....?	$11 \times 6 =$.....?

6.

$\frac{1}{2}$ of 320 =.....?	$\frac{1}{10}$ of 320 =.....?	$\frac{1}{16}$ of 320 =.....?
$\frac{1}{8}$ of 320 =.....?	$\frac{1}{4}$ of 320 =.....?	$\frac{1}{32}$ of 320 =.....?

7. $5 \times 320 =$.....? $4 \times 320 =$.....?

8. What part of 320 is 40? What part is 80? 20?

1. A horse can go 1 mile in 6 minutes. At this rate how many rods would he go in an hour?

2. A carriage wheel measures 12 feet around the tire. How many times will it turn around in going 3 miles? In going 5 miles?

3. A wagon wheel is 10 feet around the tire. How many yards will it move along in going around 120 times?

4. A boy's top-string is 2 yards long. He cuts from it a piece 18 inches long. What is the length in feet of the remaining part?

5. A mile of gas pipe is laid at $5 a rod. What is the cost of laying?

6. What will 4 rods of hose cost at 25¢ a foot?

7. What will be the cost of a ditch half a mile long at $1.25 a rod?

8. What is the distance around a lot which is 50 yards and 2 feet long and 8 yards and 1 foot wide?

9. One cord of wood costs $5½. How much does 4 cords cost?

10. How many bins holding 5½ bushels can be filled from 16½ bushels of grain?

11. A man walked 16½ miles 1 day and 5½ miles the next. How many miles did he walk in all? How many miles farther the first day than the second?

12. At $5½ a ton, how many tons of coal can be bought for $22?

13. A farmer bought 16 sheep at $3 each and sold them at the rate of 3 for $12. Find the entire cost. Find how much he received. Find his gain.

14. What is the cost of a bale of cotton, containing 500 pounds, at 8½¢ a pound? At 10½¢ a pound?

1. Draw diagrams on a scale of 1 inch to 3 feet for rooms of the following dimensions:

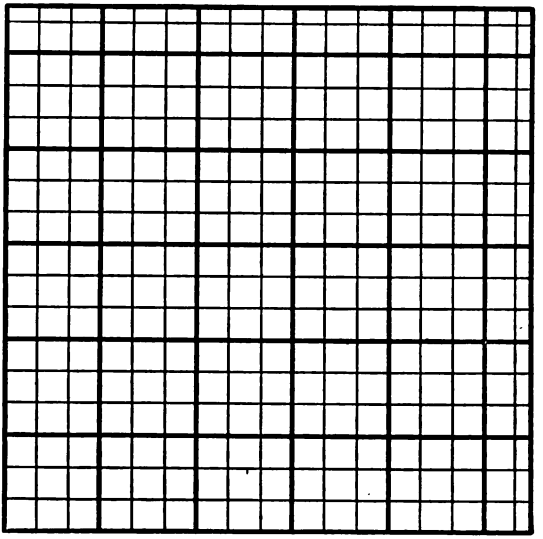
12 feet long and 9 feet wide
24 feet long and 18 feet wide
30 feet long and 27 feet wide
33 feet long and 21 feet wide
15 feet long and 12 feet wide
27 feet long and 15 feet wide
24 feet long and 15 feet wide
Give the perimeters in feet. In yards.

2. Draw diagrams on a scale of 1 inch to 3 yards for lots of the following dimensions:

15 yards long and 12 yards wide
18 yards long and 15 yards wide
12 yards long and 6 yards wide
15 yards long and 9 yards wide
24 yards long and 12 yards wide
33 yards long and 15 yards wide
27 yards long and 12 yards wide
Give the perimeters in yards. In feet.

3. Draw diagrams on a scale of $\frac{1}{2}$ inch to $\frac{1}{8}$ mile for fields of the following dimensions:

$\frac{1}{2}$ mile long and $\frac{1}{4}$ mile wide
1 mile long and $\frac{1}{2}$ mile wide
2 miles long and $1\frac{1}{2}$ miles wide
$1\frac{1}{2}$ miles long and $\frac{1}{2}$ mile wide
$\frac{3}{4}$ mile long and $\frac{1}{2}$ mile wide
$1\frac{1}{2}$ miles long and $\frac{3}{4}$ mile wide
$1\frac{1}{4}$ miles long and $\frac{3}{8}$ mile wide
Give the perimeters in miles. In rods.



Plan of a square floor, drawn to a scale of ½ inch to 1 yard.

1. Measure the figure. What is the length? The width?

2. How many feet long is the floor? How many feet wide? How many yards wide?

3. Count the number of square yards. By what name is $16\frac{1}{2}$ feet known? By what name is $5\frac{1}{2}$ yards square known?

4. If a square were 320 times as long and wide as the square that the figure represents, how long and wide would it be? What is such a square figure called?

5. Draw a 6-inch square. Divide it into half-inch squares. How many squares are there in one row? How many rows? How many squares are there in all?

6. How many square inches in a square foot?

7. How many square feet in a square yard?

8. How many square yards in a square rod?

1. How many square feet in one row on one side of a square yard? How many rows of the same number of square feet? How many square feet in a square yard?

2. How many square yards in a square rod? (See p. 14.)

3. How many square yards in 2 square rods?

4. How many square yards in 4 square rods?

5. How many square yards in one-half of a square rod?

6. How many square yards in $2\frac{1}{2}$ square rods?

7. A garden contained $2\frac{1}{2}$ square rods and was 1 rod wide. How long was it?

8. A lot contains 20 square rods. How many square yards are there in it?

9. A lot containing 80 square rods was sold for \$240. What was the price per square rod?

10. A lot containing 160 square rods was sold at the rate of \$3 a square rod. How much was received for it?

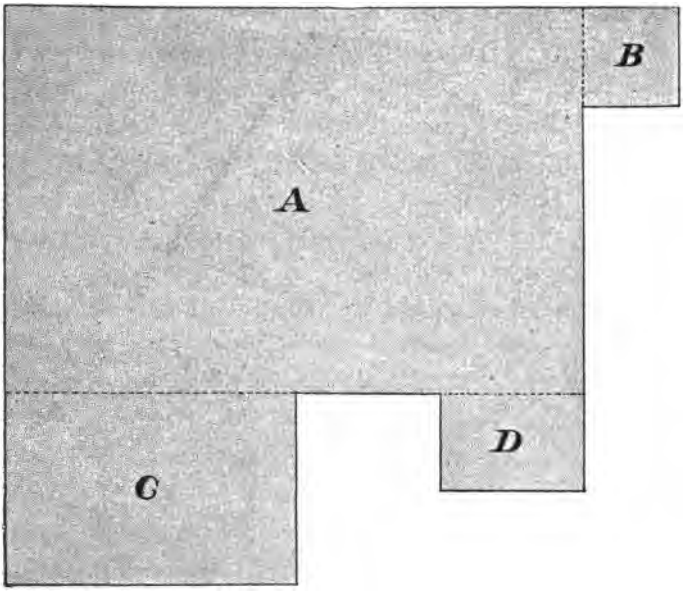
11. A street in a village was 6 rods wide and 40 rods long. How many square rods did it contain? How many square yards?

12. A street 250 rods long contains 10 blocks. Not counting crossings, how many rods long is each block? How many yards long is each block? How many feet?

13. Find the cost at \$2 a sq. yd. of paving 60 yd. of length of a street with a pavement 3 rd. wide. Find the cost of a strip 1 yd. wide reaching across the street.

14. A street is 28 blocks long. Each block, including crossing, is 15 rods long and 5 rods wide. How many square rods does the street contain?

15. A field 16 rods long and 10 rods wide contains how many square rods? What is the perimeter in rods? In yards? In feet?



Plan of a garden, drawn to a scale of 1 inch to 12 feet.

To find the area of the entire garden, first divide as indicated by the dotted lines.

1. The area of B is equal to how many square feet? How many square yards?

Find the area of D in square feet. In square yards. Find the area of C.

3. Find the area of A in square feet. In square yards.

4. Find the area of the entire garden in square yards.

5. C equals what part of A? D equals what part of C? Of A? B equals what part of D? Of C? Of A?

6. If 1 inch on the plan represents 4 yards, what is the area of each section in square yards? In square feet?

7. If 1 inch on the plan represents 8 rods, what is the area of each section in square rods?

TABLE OF SQUARE MEASURE

144	sq. in. (square inches)	= 1 sq. ft. (square foot).
9	sq. ft. (square feet)	= 1 sq. yd. (square yard).
30¼	sq. yd. (square yards)	= 1 sq. rd. (square rod).
160	sq. rd. (square rods)	= 1 A. (acre).
640	A. (acres)	= 1 sq. mi. (square mile).

1. How many square rods in 60 acres?
2. In 32 sq. rd. how many square yards?
3. How many sq. ft. in 63 square yards?
4. In 37 sq. ft. how many square inches?
5. In 56 A. and 13 sq. rd. how many square rods?
6. In 47 sq. rd. and 9 sq. yd. how many square yards?
7. In 37 sq. yd. and 7 sq. ft. how many square feet?
8. Add:

4 sq. ft., 43 sq. in.	14 sq. yd., 2 sq. ft.
6 sq. ft., 84 sq. in.	11 sq. yd., 4 sq. ft.
7 sq. ft., 17 sq. in.	4 sq. yd., 3 sq. ft.

9. Add:

16 sq. rd., 4 sq. yd.	60 A., 15 sq. rd.
47 sq. rd., 8 sq. yd.	137 A., 40 sq. rd.
64 sq. rd., 15 sq. yd.	256 A., 56 sq. rd.
32 sq. rd., 3¼ sq. yd.	186 A., 49 sq. rd.

10. Subtract:

18 sq. ft., 56 sq. in.	16 sq. yd., 7 sq. ft.	18 A., 86 sq. rd.
5 sq. ft., 13 sq. in.	11 sq. yd., 3 sq. ft.	5 A., 17 sq. rd.

11. Multiply:

6 sq. ft., 24 sq. in.	6 sq. yd., 3 sq. ft.	3 A., 80 sq. rd.
6	3	2

1. A board is 13 feet and 4 inches long and 12 inches wide. What is the area of one side in square feet?

2 A house 24 feet wide covers 72 square yards of ground. How long is it? What is the distance around it?

3. The blackboards in a schoolroom are equal to 1 blackboard 54 ft. long and $4\frac{1}{2}$ ft. wide. How many sq. ft. of surface in all of the blackboards? How much will it cost to slate them at 36¢ a square yard?

4. How many acres of ground in 4000 square rods?

5. How much will a farm 240 rods long and 60 rods wide cost at $35 an acre?

6. A man has 10 acres and 90 square rods. He buys 8 acres and 70 square rods. How much land does he then have?

7. How many square feet in the floor of a room that is 24 feet long and $12\frac{1}{2}$ feet wide? How much will it cost to paint the floor at 25¢ a square yard?

8. How many square yards of cloth will it take to cover a table that is 48 inches long and 36 inches wide?

9. The floor of a hall 36 feet long and 6 feet wide is paved with marble blocks 1 foot square. How many blocks did it take to pave the hall?

10. A hall 24 feet long and 6 feet wide is paved with tiles 6 inches square. How many tiles are required?

11. A room is 18 feet long, 15 feet wide, and 9 feet high. How many square yards in the floor and ceiling. How many square yards in the walls?

12. How many square feet of sidewalk in 9 blocks of 528 feet each, if the sidewalk is 6 feet wide?

13. A building is 150 feet long and $40\frac{1}{2}$ feet wide. How many square yards does it cover?

1. Divide:

264 by 11, by 21, 31, 41, 51.
528 by 12, by 22, 32, 42, 52.
377 by 13, by 23, 33, 43, 53.
434 by 14, by 24, 34, 44, 54.
675 by 15, by 25, 35, 45, 55.
352 by 16, by 26, 36, 46, 56.

2. Divide each of the following numbers by 13, 14, 16, 29, 38, 46, 57, 74, 88, and 97:

118	154	172	120	156
213	237	248	253	269
3123	3234	3345	3456	3567
4678	5788	6879	7890	8901
9012	8123	7243	6542	5987

3. A man paid \$31.46 for eggs at 13¢ a dozen. How many dozen did he buy?

4. A tank holds 300 barrels of water. When it is $\frac{1}{15}$ full, how many barrels does it hold?

5. In a flock of 1736 sheep, $\frac{1}{14}$ of the number were lambs. How many lambs were there?

6. A man bought 20 feet of piping for \$6.40. How much per foot did it cost?

7. A man bought 18 yards of cloth for \$36.72. How much did it cost a yard?

8. A grocer sold 19 pounds of butter for which he received \$4.37. How much per pound did he get?

9. A boy can ride 15 miles an hour on his bicycle. At the same rate how many hours would it take him to ride 5136 miles?

10. The schoolroom floor contains 1100 square feet and is 25 feet wide. How long is it?

LONG DIVISION

1. Solve the following:

(1) $21\overline{)1302}$ (5) $34\overline{)7242}$ (9) $75\overline{)24300}$ (13) $121\overline{)9882}$

(2) $21\overline{)1575}$ (6) $37\overline{)4514}$ (10) $81\overline{)9801}$ (14) $124\overline{)10584}$

(3) $22\overline{)5302}$ (7) $37\overline{)8621}$ (11) $86\overline{)18404}$ (15) $214\overline{)18490}$

(4) $34\overline{)4216}$ (8) $62\overline{)7626}$ (12) $94\overline{)29610}$ (16) $315\overline{)29704}$

2. A field of 120 acres of corn yielded 4560 bushels. What was the yield per acre?

3. A man paid \$8400 for a farm that cost him \$75 per acre. How many acres were there in the farm?

4. A man paid \$11,200 for town lots, at the average price of \$175 a lot. How many lots did he buy?

5. A field is 140 rods long and contains 11,900 square rods. How many rods wide is the field?

6. How many cars that hold 48 tons each will be required to hold 6144 tons of coal?

7. There are 144 square inches in a square foot. How many square feet are there in 19,008 square inches?

8. How many bushels are there in a load of corn that weighs 3458 pounds, counting 56 pounds to the bushel? Write the remainder over the divisor as a fraction in the result.

9. Solve the following, writing the remainders over the divisors, as is shown in the first:

(1) $27\overline{)1170}$ $43\frac{9}{27}$ (4) $123\overline{)15293}$ (7) $280\overline{)18840}$ (10) $480\overline{)78060}$

(2) $64\overline{)3120}$ (5) $165\overline{)12595}$ (8) $275\overline{)30910}$ (11) $300\overline{)19410}$

(3) $123\overline{)9307}$ (6) $208\overline{)19864}$ (9) $424\overline{)47223}$ (12) $375\overline{)45600}$

CRANBERRY RAISING

State	Acres.	Bushels.	Bushels per acre.	State	Acres.	Bushels.	Bushels per acre.
Nebraska	1	20	20	Indiana	70	4360	$62\frac{20}{70}$
South Dakota	1	22	..	Maine	90	1554	....
Kansas	1	30	..	New York	113	10877	....
Illinois	1	53	..	Michigan	150	3884	....
Iowa	1	61	..	Connecticut	275	6921	....
Washington	5	138	..	Rhode Island	300	6559	....
Oregon	6	712	..	Massachusetts	5128	598906	....
Minnesota	22	1120	..	Wisconsin	5821	111098	....
New Hampshire	22	973	..	New Jersey	8356	230221	....

The table shows the number of acres cultivated and the total number of bushels of cranberries grown in the states named in the year 1900.

1. Find the yield in bushels per acre for Washington. For Oregon. For Minnesota.

NOTE.—When there is a remainder write it over the divisor and make the fraction, thus formed, a part of the quotient.

$$\begin{array}{r} 50\frac{20}{22} \\ 22\overline{)1120} \\ 110 \\ \hline 20 \end{array}$$

Answer, $50\frac{20}{22}$ bushels per acre.

A zero is put in the quotient, because 22 does not go into 20 a whole number of times.

Show that this answer is about 51.

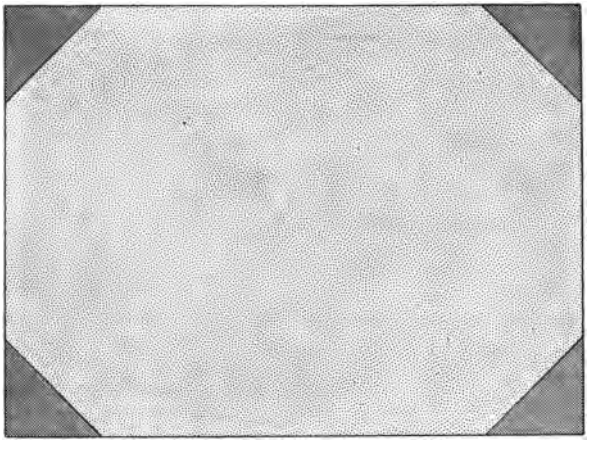
2. Find the yield per acre for each of the other states.

3. Divide the sum of the total yields by the sum of the total number of acres of the 9 states in the first part of the table, and find the average yield per acre.

4. Find the average yield for the first 5 states.

5. Find the average yield per acre for Illinois, Iowa, Indiana, and Minnesota together.

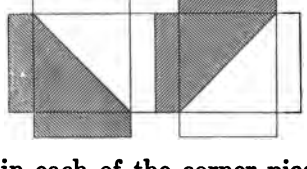
6. Find the average yield per acre for New York, Massachusetts, and Oregon together. Find the average yield per acre for Connecticut and Rhode Island together.



1. This drawing stands for a blotting pad, 24 in. long and 18 in. wide. How long is the drawing? How wide? What is the scale of the drawing?

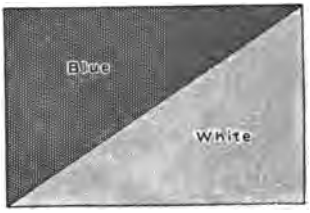
2. The corners are bound with pieces of red leather as shown. How long are the short sides of the corner pieces on the drawing? How long on the pad itself?

3. Cut two triangles out of paper and make them the size of a corner piece which can be seen from the top of the pad. How many square inches are there in each of them?

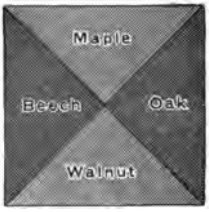


4. There is a strip, for pasting, one inch wide, on each of the two short sides of the leather corner pieces. How many square inches are there in each of the corner pieces with its two strips?

5. How long and how wide is the smallest piece of leather out of which the four corner pieces can be cut, each having two strips for pasting, as shown?

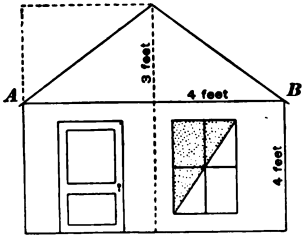


1. A rectangular flag, 6 feet long and 4 feet wide, is made of 2 pieces of cloth, sewed together on a line, as shown here. What part of the flag is blue? How many square feet are there in the white part?



2. The top of a box, 8 inches square, is made of 4 different kinds of wood, as shown. Draw, and cut out of paper, four pieces equal to the different pieces of wood. How many square inches are there in each piece? In all?

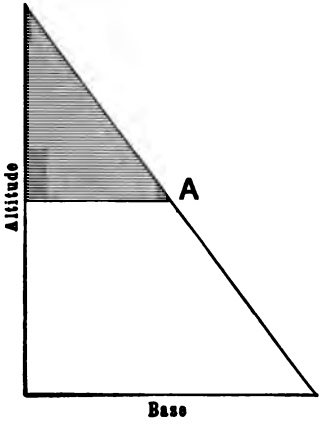
3. A rectangular garden, 24 feet by 40 feet, was separated into two parts by a straight line from corner to corner (a diagonal). Make a scale drawing of the garden in which one inch stands for 8 feet. How many square feet are there in each part of the garden?

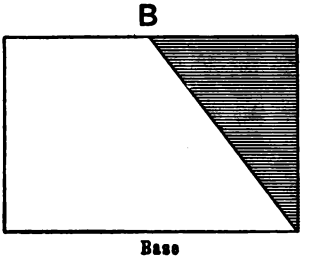


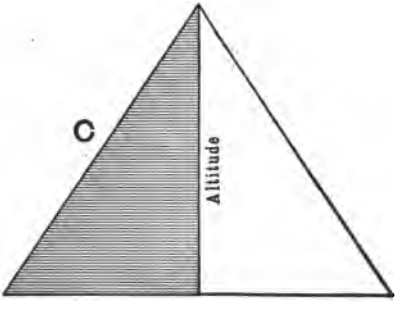
4. The front end of Harry's playhouse is 8 feet wide, 4 feet high on the sides, and 7 feet high at the centre. Make a scale drawing of the end.

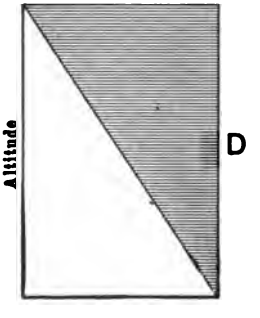
5. How many square feet are there in the front end below the line AB? Above AB? In the entire front end of the playhouse?

6. A front window is 3 feet high and 2 feet wide. A curtain covers a part of the window, as shown in the picture. How many square feet of the window does it cover?


Altitude
A
Base


B
Base


C
Altitude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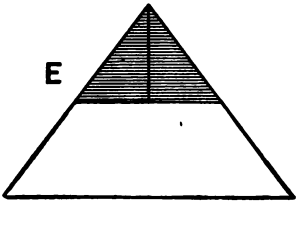
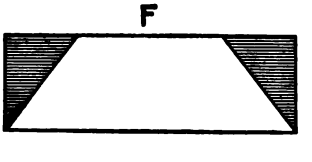
D


E


F

1. What is the altitude of figure A, page 24? The base?

2. Find the middle points of the altitude and of the long side of the triangle. Suppose the triangle cut on a straight line passing through these two points, and the parts turned about and fitted side by side, as in B.

3. What kind of figure is thus made (see B)? What is the length of B? The width? The area? The area of triangle, A?

4. Cut a 4-inch square. Draw a line connecting two opposite corners, and on this line cut the square in two. How long is the altitude of one of the triangles thus made? The base? Find the area.

The side a triangle is supposed to rest on, is called the *base*. The shortest distance from the base to the highest point is the height, or *altitude*.

5. What are the dimensions (base and altitude) of the triangle, C? Suppose the left side of the triangle, C to be turned about and laid upon the longest side of C, as in the figure D. What is the figure D?

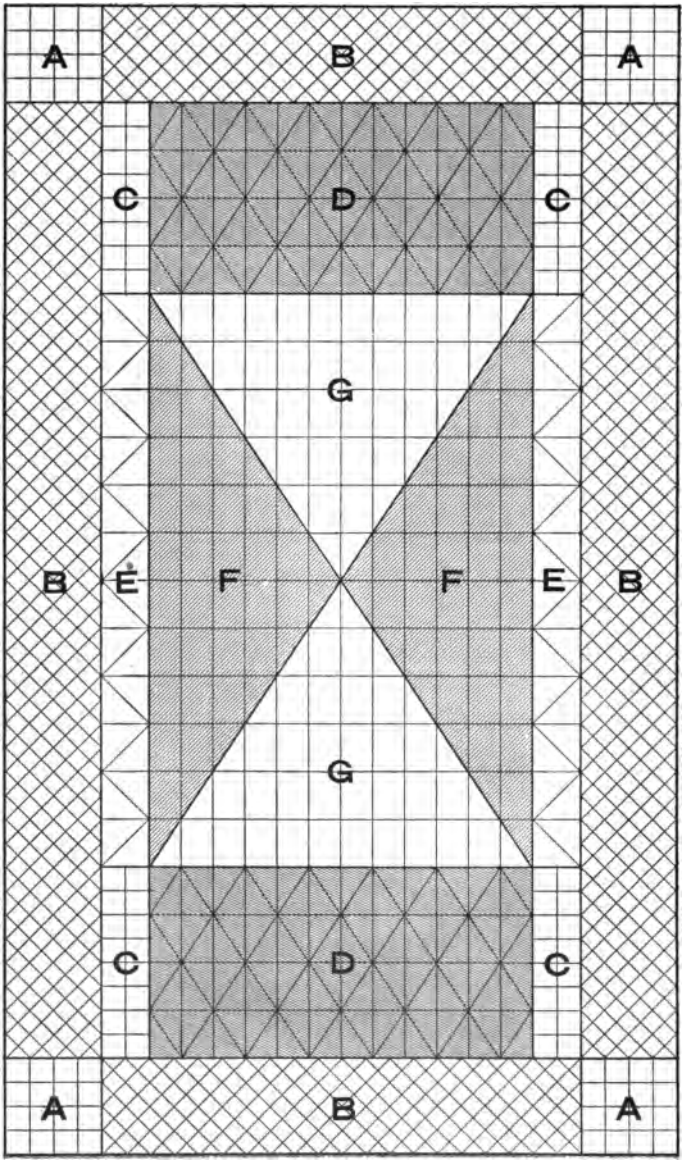
6. What are the dimensions of D? What is the area?

7. What is the area of the triangle, C?

8. Suppose the base of the triangle, C to be 8 feet and the altitude, 8 feet, what is the area of the triangle?

9. The area of such a triangle as C may be found in another way, as shown in figures E and F.

10. Find the middle points of the two sides of triangle, E. Suppose the triangle to be cut through them and the top part equally divided and placed on the sides of the lower part of the triangle, as in the figure F. What kind of figure is thus formed? What are the dimensions? What is the area? What is the area of triangle, E?


A
B
A
C
D
C
G
B
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E
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D
C
A
B
A

1. Page 26 shows a drawing to the scale, 1 in. to 2 feet. It is the picture of the tiling in a small hall. How long is the hall? How wide is it? What is the area?

2. The cost of tiling the 4 corners, A, was $1.80. How much per square foot was the cost?

3. The cost of tiling one short outside border, B (without the corners), was $2.40. How much was that per square foot? At that rate, what was the cost of tiling the whole outside border, B?

4. The cost of tiling the corners, C, was $1.30 for each corner. Find the cost per square foot? How much was the cost for the 4 corners?

5. How many square feet in D? In the two D's?

6. At 55 cents a square foot, how much did it cost to tile D? The two D's?

7. The cost of tiling one inside border, E, was $2.88. How much was the cost per square foot?

8. What did the tiling for the two E's cost?

9. What is the area of the tiling for F? For G?

10. The cost of tiling F was $2.70. How much was the cost per square foot? At that rate, what was the cost of tiling the two F's?

11. The cost of tiling G was $3.12. How much was the cost per square foot? At that rate, what did it cost to tile the two G's?

12. What was the cost of tiling the two F's and the two G's together?

13. What was the cost of tiling the two E's, the two D's, and the four C's?

14. What was the cost of tiling this whole hall?

A multiplication table for larger numbers than 12 is not needed. This will be seen by the following:

$2\times13=2$ 10's and 2 3's $=20+6=26$

$4\times14=4$ 10's and 4 4's $=40+16=56$

$3\times15=30+15=45$

This work should be done mentally, the pencil being used only to write the final product.

1. Solve mentally:

$3\times13=?$	$6\times13=?$	$9\times13=?$
$4\times13=?$	$7\times13=?$	$10\times13=?$
$5\times13=?$	$8\times13=?$	$11\times13=?$

2. Solve mentally:

$2\times14=?$	$5\times14=?$	$8\times14=?$
$3\times14=?$	$6\times14=?$	$9\times14=?$
$4\times14=?$	$7\times14=?$	$11\times14=?$

3. How many are 26 and 10? 26 and 20? 26 and 30? 36 and 10? 39 and 30? 49 and 30? 49 and 20? 49 and 40? 42 and 20? 42 and 30? 42 and 50? 42 and 40? 65 and 20? 65 and 40? 65 and 30? 65 and 60? 78 and 10? 67 and 10? 78 and 20? 78 and 30? 84 and 10? 84 and 30? 59 and 10?

4. Count by 10's from 13 to 103. From 27 to 127. From 46 to 116. From 87 to 137.

5. Count by 20's from 14 to 134. From 4 to 144. From 33 to 153. From 67 to 167.

6. Count by 30's from 16 to 166. From 9 to 189. From 17 to 197. From 28 to 178.

To add 38 and 23, first add to 38 the 10's in 23, then the 1's, using pencil only to write the last result. Thus, 38 and 20 are 58 and 3 are 61.

1. Solve mentally:

26+13=?	43+15=?	34+14=?
39+13=?	42+14=?	45+15=?
56+14=?	65+13=?	60+15=?
28+14=?	75+15=?	70+14=?
48+16=?	32+16=?	90+15=?
64+16=?	43+16=?	38+15=?

2. Review problem 1, page 28. 13 equals what part of 52? Of 39? Of 78? Of 91?

3. Review problem 2, page 28. 14 equals what part of 42? Of 28? Of 56? Of 84? Of 70? Of 98?

If products like 13×2, and 14×4, are wanted, turn them mentally into 2×13, and 4×14, and then work them the same way as in problems 1 and 2, page 28.

4. Solve mentally:

13×2=?	13×6=?	14× 8=?
14×2=?	13×4=?	14×10=?
14×3=?	14×4=?	13× 8=?
13×3=?	13×5=?	13× 7=?
14×5=?	14×7=?	13×10=?
14×6=?	13×9=?	14× 9=?
15×3=?	15×4=?	15× 5=?

5. Build the table of 15's as the table of 14's was made in problem 2, page 28.

6. Build the table of 16's the same way.

7. Build the table of 20's.

8. How many stripes in our flag?

9. How many states were there when the number of stars was three times the number of stripes?

10. When will the number of stars be 4 times the number of stripes?

1. 8 times the number of stripes in our flag is the number of years from 1800, until Roosevelt was elected President. In what year was he elected President?

2. From the Declaration of Independence to the World's Fair in Chicago was 9 times as many years as there are stripes in the flag. How many years was it?

3. A man fed his horse $1\frac{1}{2}$ pecks of oats each day for 4 weeks. How many bushels of oats did he feed him?

4. Mr. Jones traveled 195 mi. a day for 2 weeks. How far did he travel in the two weeks?

5. A train ran 14 miles in 30 minutes. How far would it run at the same rate in $2\frac{1}{2}$ hours?

6. A piece of ground 1008 ft. wide is divided into 14 equal strips. What is the width of each strip?

7. A boy bought 3 dozen oranges at the rate of 15¢ a dozen. What did they cost him apiece?

8. A girl bought 12 handkerchiefs at the rate of 2 for 15¢. What did they cost her?

9. Railroad fare for a picnic excursion was 15¢ for the round trip. How much was collected on a train of 9 cars with 65 persons in each car?

10. In a square mile of land there are 16 farms of equal size. How many acres in each (see page 17)? If one of these farms is divided into 3 fields, two of which contain 16 acres each, what is the area of the third field?

11. From a bin containing 80 lb. of meal, two 8-lb. packages were taken. How many 16-lb. packages can be made from what is left?

12. How many 4-oz. packages can be made from eight 10-lb. packages of spices, allowing 8 oz. for loss in packing? How many 4's in 80 sixteens?

Answer all you can orally.

In the following problems, "at the same rate" is understood.

1. 3 pounds of butter cost 75¢. What is the cost of 4 pounds?

2. At 9 barrels of flour for $45, find the cost of 7 barrels.

3. 5 acres of land cost $250. What is the cost of 7 acres of land?

4. I received $63 for 9 weeks' work. What should I receive for 12 weeks' work?

5. What is the cost of a dozen chairs if $\frac{1}{4}$ of a dozen cost $12?

6. If I pay 4¢ for 10 marbles, what should I pay for 25 marbles?

7. Eggs sell at 30¢ for 2 dozen, what is the cost of $\frac{1}{3}$ of a dozen?

8. A dozen pairs of boots cost $36. What is the cost of 8 such pairs?

9. When 2 gallons of syrup cost $1.50, what is the cost of 3 quarts?

10. A man receives $1200 a year and spends $45 a month. How much does he save the first 6 months? How much does he spend?

11. A piece of string is equal to 6 lengths of an 18-inch rule. How many feet long is it?

12. A man had 30 bushels of wheat, $\frac{2}{3}$ of which he sold at 95¢ a bushel. The remainder he sold at 98¢ a bushel. What did he receive for the entire load?

13. The curtains for a room with 3 windows cost $5. What will they cost for a room with 12 windows?

14. 3 dozen neckties cost $1.80. What is the cost of 4 neckties?

NOTE—In plastering, many contractors make no deductions for windows or doors on account of the extra time necessary to do the work carefully around the frames. In all the problems, therefore, on plastering, unless otherwise stated, the walls are counted as solid.

1. A room is 9 ft. by 21 ft. and 9 ft. high. How many sq. yd. in the walls? In the ceiling? What will it cost to plaster the walls and ceiling at 21¢ a square yard?

2. A room is 12 feet square and 10 feet high. What will it cost to plaster the walls and ceiling at 23¢ per square yard?

3. What will be the cost of plastering the walls of a room 9 by 12 feet and 9 feet high at 45¢ a square yard?

4. What will be the cost of plastering the ceiling of a room 24 feet square at 28¢ a square yard?

5. Mr. Jones wishes to plaster the walls and ceilings of 3 rooms. The first room is 9 by 12 ft., the second 12 by 15 ft., and the third 15 by 18 feet. The height of each room is 9 feet. What will be the cost of plastering the three rooms at 28¢ a square yard?

6. What would be the cost of enough carpet a yard wide for the three rooms at 75¢ a yard?

7. The floor of a room contains 324 square feet. One side of it is 4 yards long. How many yards long is the other side?

8. The 4 walls of a square room 8 feet high contain 384 square feet. What is the length and the width of the room?

9. What is the length of a wall 12 feet high the area of whose side is 3 times 264 square feet?

10. A room is 10 feet wide and $11\frac{1}{2}$ feet long. A rug on the floor is $2\frac{1}{2}$ yards wide and 3 yards long. How much of the floor is not covered by the rug?

1. Solve mentally:

(1) $23+10=?$	(5) $54+20=?$	(9) $47+50=?$
(2) $47+20=?$	(6) $65-20=?$	(10) $74-40=?$
(3) $54-10=?$	(7) $83-30=?$	(11) $67+30=?$
(4) $65+20=?$	(8) $63+30=?$	(12) $97-63=?$

2. Solve mentally, by adding, or subtracting the 10's, then the 1's:

(1) $36+12=?$	(5) $67-34=?$	(9) $65-44=?$
(2) $46+23=?$	(6) $55+24=?$	(10) $87-43=?$
(3) $57+32=?$	(7) $55+34=?$	(11) $62+37=?$
(4) $74-22=?$	(8) $65-34=?$	(12) $68-47=?$

3. Solve mentally, as rapidly as you can give results correctly; first adding, or subtracting the 10's, then the 1's:

(1) $38+22=?$	(5) $38-22=?$	(9) $65-17=?$
(2) $38+32=?$	(6) $45+16=?$	(10) $65-27=?$
(3) $38+43=?$	(7) $45+26=?$	(11) $62+28=?$
(4) $38+54=?$	(8) $45+37=?$	(12) $62+38=?$

4. How many 2's in 4? How many 2's in 2 fours? In 3 fours? In 4 fours? In 5 4's? In 6 4's?

5. How many 4's in 8? In 2 8's? In 3 8's? In 4 8's? In 6 8's? In 7 8's? In 10 8's?

6. How many 3's in 6? In 2 6's? In 3 6's? In 5 6's? In 7 6's? In 12? In 2 12's? In 3 12's?

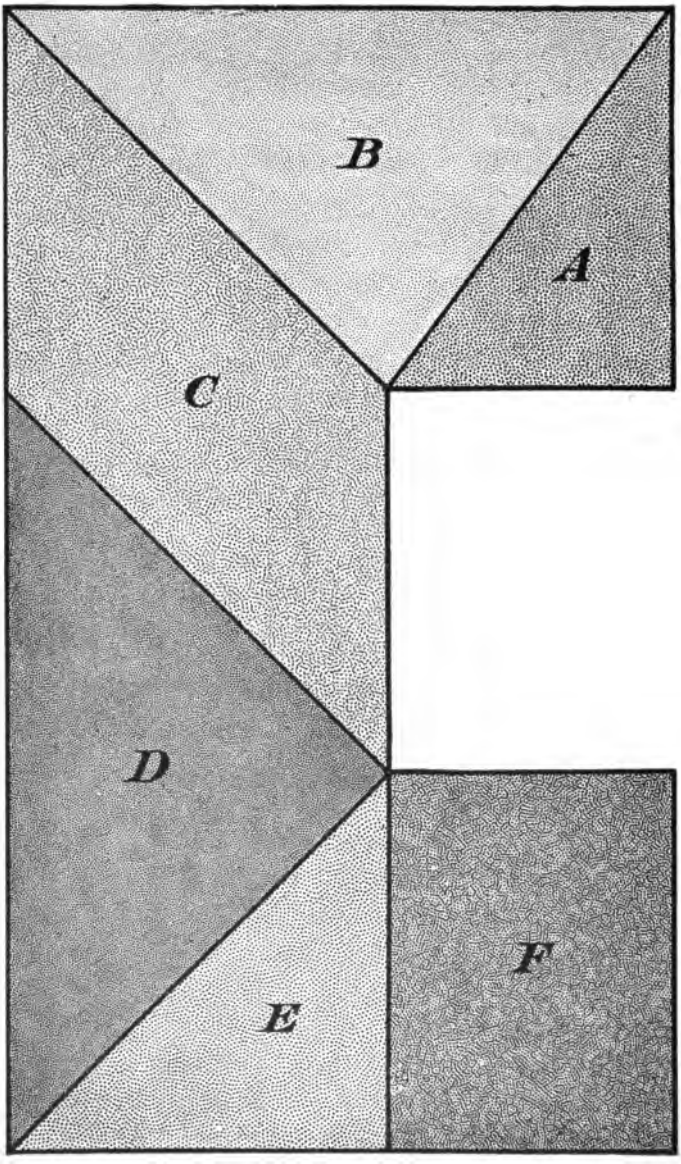
7. How many 5's in 10? In 2 10's? In 3 10's? In 4 10's? In 15? In 2 15's? In 5 15's?

8. How many 6's in 12? In 2 12's? In 3 12's? In 5 12's? In 18? In 2 18's? In 3 18's?

9. How many 8's in 24? In 2 24's? In 4 24's?

10. How many 7's in 14? In 2 14's? In 3 14's?

11. How many 8's in 40? In 2 40's? In 5 40's?


B
A
C
D
F
E

The plan on the opposite page represents a garden, which has been cut into various parts by the walks running through it. The plan is drawn to the scale of 1 inch to 10 feet.

1. Measure the line around the entire plan and find the length of the path around the garden.

2. How many square feet does A represent? B? C?

3. How many square feet does D represent? E? F?

4. How many square feet are there in the entire garden?

5. A is what part of B? Of C? Of D? Of E? Of F?

6. F is what part of the entire garden?

7. How did you find the area of C?

8. How did you find the area of D?

9. In what other ways beside finding the areas of the triangles could you find the area of the entire garden?

Suppose the plan on the opposite page, representing a piece of land, is drawn to a scale of 1 inch to 8 rods.

10. What is the value of A at 25¢ a square rod?

11. What is the value of B at 30¢ a square rod?

12. What is the value of C at 28¢ a square rod?

13. What is the value of D at 27¢ a square rod?

14. What is the value of E at 26¢ a square rod?

15. What is the value of F at 29¢ a square rod?

16. How many sq. rd. in the entire field?

17. How many acres?

18. How many sq. yd. in A?

19. How many sq. yd. in B?

20. How many sq. yd. in C?

21. How many sq. yd. in D?

22. How many sq. yd. in E?

23. How many sq. yd. in F?

1. A woman who raises chickens put 13 eggs under each of 6 hens. The first hatched out all but 1, the second all but 2, the third all but 3, the fourth all but 4, and the others all but 5 each. How many chickens were hatched?

2. How many pigeons will it take to pick up a bushel of corn (56 lb.) if each one picks up 4 ounces?

3. A man buys 12 tons of hay for $80 and sells it for 60¢ per cwt. How much does he make?

4. An expressman receives $3.25 per day for 30 days. It costs him 30¢ per day to feed his horse, and $4.20 for repairs to his wagon. How much has he left?

5. An express company carries 400 packages at 15¢ each; 28 trunks at 50¢ each, and 12 bicycles at 40¢ each. How much does the company get for all?

6. There are 40 street cars on one line and each can carry 60 people. How many people do they all carry in 12 round trips if $\frac{1}{2}$ carry their full number each way and the remainder carry 30 persons each way?

7. A baker has 400 loaves of bread. He sells $\frac{3}{4}$ at 5¢ per loaf, 60 loaves at 4¢, and gives the rest away. How much does he get for the bread?

8. How many sheep must there be to produce a ton of wool if the wool from each sheep weighs 4 pounds? If each sheep produces 5 pounds?

9. A farmer raised 840 bu. of potatoes on 5 acres of land. What was the average value of the potato crop per acre at 40¢ per bushel?

10. A man set out 12,000 cabbage plants, but $\frac{1}{6}$ of them died and $\frac{1}{10}$ of the remainder were blighted. What did he receive for the rest at $3 per hundred?

1. An orchard of 600 trees produced 3 bbl. of apples to the tree. The owner sold them at $1.40 per bbl., but the barrels cost him 25¢ each. What did he get for the apples after paying for the barrels?

2. An orchard produced 210 bu. of peaches. If 1 bu. fills 5 baskets, what is the value of the crop at 20¢ a basket?

3. A man pays 1¢ per box for picking berries, and $\frac{1}{4}$¢ each for boxes. If he sells 1000 boxes of berries for $60.00, what is his profit?

4. A boy's pay for a week's work at berry-picking at 1¢ per box was $5.40. How many boxes did he pick daily during the six days if he picked the same number each day?

5. A farmer sells 1000 boxes of berries to a city grocer at 6¢ a box. The grocer pays $5.00 to get them to the city and sells them at 8¢ a box. What is his gain?

6. A woman who kept chickens bought 12 bu. of feed for them at 35¢ per bu. She sold 120 doz. eggs at $12\frac{1}{2}$¢ a doz. and 40 chickens at 25¢ each. How much more did she receive than she paid out?

7. How many chickens averaging 5 lb. each and worth 6¢ per lb. can be bought for $75?

8. If 15 chickens are worth as much as 1 sheep, and 6 sheep are worth as much as 1 cow costing $28.80, what is 1 chicken worth?

9. A carpenter builds a fence for $56. The lumber costs him $15 and he pays each of three men $2.75 per day for 4 days. What is his share of the $56?

10. A man earns $1.50 every day he works and pays 50¢ a day for his board. If he works only 16 days during the month of May, how much has he left after paying his board for the whole month?

BEET-SUGAR PRODUCTION OF THE UNITED STATES

STATE	1903			1902		
	Factories	Acres Sown	*Tons of Sugar	Factories	Acres Sown	*Tons of Sugar
New York	2	7,000	4,479	2	6,500	2,799
Wisconsin	1	5,800	4,911	1	3,400	3,463
Ohio	1	2,500	2,009	1	2,450	1,473
Michigan	20	117,100	57,064	16	98,009	48,848
Minnesota	1	3,800	3,125	1	4,500	3,054
Nebraska	3	11,400	8,869	3	9,980	9,430
Colorado	8	52,800	39,566	5	39,449	34,623
Utah	7	18,700	20,670	6	18,600	16,987
Oregon	1	1,800	1,250	1	3,100	2,025
Washington	1	4,400	2,213	1	2,300	1,641
Idaho	1	5,300	3,571			
California	7	62,195	60,608	7	71,234	71,120

*Ton = the long ton of 2240 pounds.

1. Find for 1902 the total number of factories; of acres sown; and of tons of sugar. Find the same for 1903.

2. Find for each state from 1902 to 1903 the increase or decrease of factories; of tons of sugar; of acres sown.

3. Find total increase from 1902 to 1903 of factories; of tons of sugar; of acres sown.

4. Find the average number of tons of sugar, and of acres sown for each New York factory in 1902. In 1903.

5. Find similar averages for Colorado. For Michigan.

6. In Kansas the total yields for each year from 1901 to 1906 of beets used for sugar, were: 1747 tons, 4251 tons, 696 tons, 6380 tons, 8606 tons, and 70,200 tons. Find the yearly increase or decrease.

7. The ton of problem 6 is the short ton of 2000 pounds. Express the results in pounds. In long tons.

8. Make other problems on the table. Current newspaper prices of sugar will suggest original problems.

	Born	Died
Paul Revere	Jan. 1, 1735	1818
Benj. Franklin	Jan. 17, 1706	1790
Wm. McKinley	Jan. 29, 1843	1901
George Bancroft	Oct. 3, 1800	1891
Abraham Lincoln	Feb. 12, 1809	1865
Charles Dickens	Feb. 7, 1812	1870
Geo. Washington	Feb. 22, 1732	1799
James R. Lowell	Feb. 22, 1819	1891
H. W. Longfellow	Feb. 27, 1807	1882
Andrew Jackson	Mar. 15, 1767	1845
Rosa Bonheur	Mar. 22, 1828	1899
Hans C. Anderson	April 2, 1805	1875
Washington Irving	April 3, 1783	1859
W. H. Prescott	May 4, 1796	1859
Audubon	May 4, 1780	1851
Queen Victoria	May 24, 1819	1901
Patrick Henry	May 29, 1736	1799

1. What is the month and day of your birth?

2. In what year were you born?

3. How old are you to-day?

4. Find how many years each person in the above table lived.

5. Which was the older, and how much older, George Bancroft or Washington Irving? James Rüssell Lowell or Henry W. Longfellow? Abraham Lincoln or Patrick Henry?

6. How many years ago (now) was each person born?

7. The birthday of William McKinley occurs how many days after that of Benjamin Franklin?

8. The birthday of Henry W. Longfellow occurs how many days before that of Andrew Jackson?

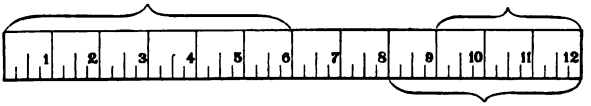
9. How many days between George Washington's birthday and that of Patrick Henry?

10. How many years, months, and days have passed since Queen Victoria was born?

11. Answer the same question for Benjamin Franklin; James Russell Lowell; Rosa Bonheur.

12. Longfellow wrote the Psalm of Life in 1838; Evangeline in 1847; Hiawatha in 1855. How old was he at the time of writing these poems?

13. Make problems using the dates of births of pupils in your class.



FOOT-RULE TO A SCALE OF 3 TO 12, OR $\frac{1}{4}$.

Answer all you can orally.

1. What part of a foot is 6 inches? 3 inches? 4 inches? 9 inches? 2 inches?

2. What part of a foot is 1 inch? 8 inches? 10 inches? 5 inches? 7 inches?

3. What part of a quart is a pint? What part of a pint is a gill? 2 gills? 3 gills?

4. What part of a yard is a foot? 2 feet? 3 feet? 4 feet? 5 feet? 6 feet?

5. What part of a gallon is a quart? 2 quarts? 3 quarts? 4 quarts? 5 quarts?

6. What part of a bushel is a peck? 2 pecks? 3 pecks? 4 pecks? 5 pecks?

7. What part of a gallon is a pint? 2 pints? 3 pints? 4 pints? 5 pints? 6 pints?

8. What part of a peck is a quart? 2 quarts? 3 quarts? 4 quarts? 5 quarts? 6 quarts?

9. What part of a yard is 6 inches? 3 inches? 2 inches? 1 inch? 12 inches? 24 inches?

10. What part of 36 is 6? 3? 2? 12? 18? 4? 9?

11. What part of a bushel is a quart? 2 quarts? 4 quarts? 8 quarts? 16 quarts?

12. What part of 32 is 2? 4? 6? 8? 16? 12?

13. What part of a bushel is a pint? 2 pints? 4 pints? 8 pints? 16 pints? 32 pints?

14. What part of 64 is 32? 16? 8? 4? 2? 24? 40? 48? 56?

1. What part of a square yard is a square foot? 2 square feet? 3 square feet? 4 square feet? 5 square feet? 6 square feet? 7 square feet? 8 square feet? 9 square feet? 10 square feet?

2. What part of a week is a day? 2 days? 3 days? 4 days? 5 days? 6 days? 7 days?

3. What part of June is 1 day? 2 days? 3 days? 4 days? 5 days? 6 days? 7 days? 10 days? 15 days? 20 days? 25 days? 24 days?

4. What part of 30 is 15? 10? 5? 3? 2? 4? 6? 12? 20? 25? 24?

5. What part of a year is a month? 6 months? 3 months? 4 months? 5 months? 8 months? 9 months? 7 months? 11 months?

6. What part of 12 is 6? 4? 3? 2? 8? 9? 10? 5? 7?

7. How many weeks are there in a year? (Ans., 52.)

8. What part of a year is 26 weeks? 13 weeks? $6\frac{1}{2}$ weeks? 1 week? 4 weeks? 12 weeks?

9. What part of 52 is 26? 13? 2? 4? 1? $6\frac{1}{2}$? $3\frac{1}{4}$?

10. What part of 24 is 12? 3? 4? 6? 2? 8? 1? 5? 9?

11. What part of 100 is 50? 25? 20? 10? 5? 75? 80? 40? $12\frac{1}{2}$? $33\frac{1}{3}$?

12. How many minutes make 1 hour?

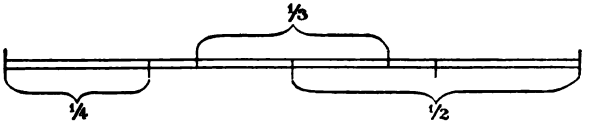
13. What part of an hour is 30 minutes? 20 minutes? 15 minutes? 10 minutes? 6 minutes? 40 minutes? 12 minutes? 24 minutes?

14. What part of 60 is 30? 20? 15? 10? 6? 40? 12? 24?

15. How many yards make 2 rods? 6 rods? 10 rods?

16. What part of 2 rods is 1 yard? 3 yards? 6 yards? 9 yards? 10 yards?

17. What part of 11 is 1? 3? 6? 9? 10?



1. Show from the drawing that $\frac{2}{2} = 1$ whole, or 1.

2. Show from the drawing that $\frac{3}{3} = 1$; that $\frac{4}{4} = 1$.

3. Show that $\frac{2}{4} = \frac{1}{2}$; that $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$; that $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

4. Show that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$; that $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$.

5. Show that $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$; that $\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$.

6. Show that $\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$; that $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

7. Show that $1 - \frac{1}{2} = \frac{1}{2}$; that $1 - \frac{1}{3} = \frac{2}{3}$; that $1 - \frac{2}{3} = \frac{1}{3}$.

8. Show that $1 - \frac{1}{4} = \frac{3}{4}$; that $1 - \frac{2}{4} = \frac{1}{2}$; that $1 - \frac{3}{4} = \frac{1}{4}$.

9. Show that $1 - \frac{2}{2} = 0$; that $1 - \frac{3}{3} = 0$; that $1 - \frac{4}{4} = 0$.

10. Show, by dividing a line into equal parts, that $\frac{1}{2} = \frac{4}{8}$; that $\frac{1}{4} = \frac{2}{8}$; that $\frac{3}{4} = \frac{6}{8}$.

11. Show, by the same divided line, that $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$; that $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$.

12. Show that $\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$; that $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$; that $\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$.

13. Show that $\frac{3}{4} - \frac{3}{8} = \frac{3}{8}$; that $\frac{1}{2} - \frac{3}{8} = \frac{1}{8}$; that $\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$.

14. In $\frac{2}{4}$, divide both the 2 and the 4 by 2, and make a fraction of the results. How does this fraction compare with $\frac{2}{4}$? Which fraction seems the simpler, $\frac{2}{4}$, or $\frac{1}{2}$?

15. In $\frac{3}{6}$, divide both the 3 and the 6 by 3, and tell how the fraction, made from the results, compares with $\frac{3}{6}$. Which fraction seems the simpler, $\frac{3}{6}$, or $\frac{1}{2}$?

1. In $\frac{4}{8}$, divide both the 4 and the 8 by 2, and tell how the fraction, made from the results, compares with $\frac{4}{8}$. Which seems the simpler, $\frac{4}{8}$, or $\frac{2}{4}$?

2. In $\frac{4}{8}$, divide both the 4 and the 8 by 4, and tell how the fraction, made from the results, compares with $\frac{4}{8}$.

3. In $\frac{6}{8}$, divide both the 6 and the 8 by 2, and tell how the fraction, made from the results, compares with $\frac{6}{8}$.

4. Divide both the 9 and the 12 of $\frac{9}{12}$ by 3 and tell how the fraction, made from the results, compares with $\frac{9}{12}$. Which is the simpler, $\frac{9}{12}$ or $\frac{3}{4}$?

5. Divide both the 10 and the 15 of $\frac{10}{15}$ by 5, and tell how the fraction, made from the results, compares with $\frac{10}{15}$. Which is the simpler, $\frac{10}{15}$ or $\frac{2}{3}$?

6. Divide both the 18 and the 24 of $\frac{18}{24}$ by 6, and tell how the fraction, made from the results, compares with $\frac{18}{24}$. Which is the simpler, $\frac{18}{24}$ or $\frac{3}{4}$?

7. How may a fraction, expressed with larger numbers, sometimes be expressed as a fraction with smaller numbers, *without changing its value*?

8. Express in smaller numbers the following fractions:

$$\frac{2}{8}\ \frac{4}{6}\ \frac{3}{9}\ \frac{4}{12}\ \frac{3}{12}\ \frac{4}{10}\ \frac{6}{12}\ \frac{9}{12}\ \frac{10}{12}\ \frac{7}{14}\ \frac{3}{15}\ \frac{5}{15}\ \frac{5}{10}$$

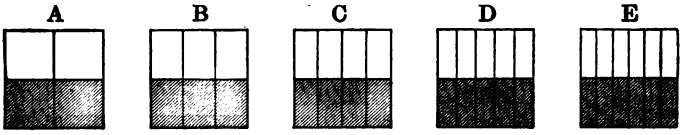
9. Express the following fractions in small numbers:

$$\frac{8}{12}\ \frac{12}{14}\ \frac{9}{18}\ \frac{9}{27}\ \frac{18}{27}\ \frac{15}{20}\ \frac{10}{20}\ \frac{14}{21}\ \frac{14}{28}\ \frac{21}{28}\ \frac{18}{24}\ \frac{20}{30}$$

Fractions are in their *simplest form*, or in their *lowest terms*, when the numbers above and below the line are the smallest possible whole numbers.

10. $\frac{10}{12}\ \frac{12}{24}\ \frac{16}{24}\ \frac{25}{30}\ \frac{30}{40}\ \frac{35}{40}\ \frac{50}{100}\ \frac{25}{100}\ \frac{75}{100}$

Express the fractions above in their lowest terms.



1. What part of A is $\frac{1}{2}$ of $\frac{1}{2}$ of A?

2. What part of B is $\frac{1}{2}$ of $\frac{1}{3}$ of B? $\frac{1}{3}$ of $\frac{1}{2}$ of B? $\frac{1}{2}$ of $\frac{2}{3}$ of B? $\frac{2}{3}$ of $\frac{1}{2}$ of B?

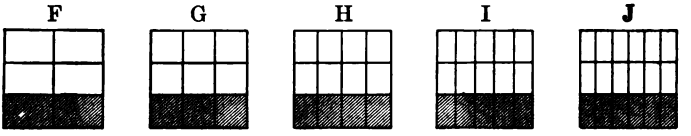
3. What part of C is $\frac{1}{2}$ of $\frac{1}{4}$ of C? $\frac{1}{4}$ of $\frac{1}{2}$ of C? $\frac{1}{2}$ of $\frac{3}{4}$ of C? $\frac{3}{4}$ of $\frac{1}{2}$ of C?

4. What part of D is $\frac{1}{2}$ of $\frac{1}{5}$ of D? $\frac{1}{5}$ of $\frac{1}{2}$ of D? $\frac{1}{2}$ of $\frac{2}{5}$ of D? $\frac{2}{5}$ of $\frac{1}{2}$ of D?

5. What part of E is $\frac{1}{2}$ of $\frac{1}{6}$ of E? $\frac{1}{6}$ of $\frac{1}{2}$ of E? $\frac{1}{2}$ of $\frac{5}{6}$ of E? $\frac{5}{6}$ of $\frac{1}{2}$ of E?

6. Show from A, B, C, D, and E that—

$$\frac{1}{2} = \frac{2}{4} \quad \frac{1}{2} = \frac{3}{6} \quad \frac{1}{2} = \frac{4}{8} \quad \frac{1}{2} = \frac{5}{10} \quad \frac{1}{2} = \frac{6}{12}$$



7. What part of F is $\frac{1}{3}$ of $\frac{1}{2}$ of F? $\frac{1}{2}$ of $\frac{1}{3}$ of F?

8. What part of G is $\frac{1}{3}$ of $\frac{1}{3}$ of G?

9. What part of H is $\frac{1}{3}$ of $\frac{1}{4}$ of H? $\frac{1}{4}$ of $\frac{1}{3}$ of H? $\frac{1}{3}$ of $\frac{3}{4}$ of H? $\frac{3}{4}$ of $\frac{1}{3}$ of H?

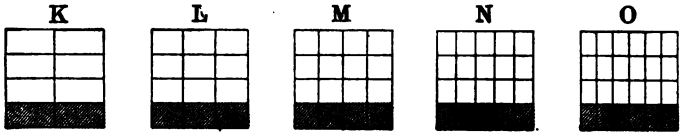
10. What part of I is $\frac{1}{3}$ of $\frac{1}{5}$ of I? $\frac{1}{5}$ of $\frac{1}{3}$ of I? $\frac{1}{3}$ of $\frac{2}{5}$ of I? $\frac{2}{5}$ of $\frac{1}{3}$ of I?

11. What part of J is $\frac{1}{3}$ of $\frac{1}{6}$ of J? $\frac{1}{6}$ of $\frac{1}{3}$ of J? $\frac{1}{3}$ of $\frac{5}{6}$ of J? $\frac{5}{6}$ of $\frac{1}{3}$ of J?

12. Show from F, G, H, I, and J that—

$$\frac{1}{3} = \frac{2}{6} \quad \frac{1}{3} = \frac{3}{9} \quad \frac{1}{3} = \frac{4}{12} \quad \frac{1}{3} = \frac{5}{15} \quad \frac{1}{3} = \frac{6}{18}$$

$$\frac{2}{3} = \frac{4}{6} \quad \frac{2}{3} = \frac{6}{9} \quad \frac{2}{3} = \frac{8}{12} \quad \frac{2}{3} = \frac{10}{15} \quad \frac{2}{3} = \frac{12}{18}$$



1. What part of K is $\frac{1}{4}$ of $\frac{1}{2}$ of K? $\frac{1}{2}$ of $\frac{1}{4}$ of K?

2. What part of L is $\frac{1}{4}$ of $\frac{1}{3}$ of L? $\frac{1}{3}$ of $\frac{1}{4}$ of L? $\frac{1}{4}$ of $\frac{2}{3}$ of L? $\frac{2}{3}$ of $\frac{1}{4}$ of L?

3. What part of M is $\frac{1}{4}$ of $\frac{1}{4}$ of M?

4. What part of N is $\frac{1}{4}$ of $\frac{1}{5}$ of N? $\frac{1}{5}$ of $\frac{1}{4}$ of N? $\frac{1}{4}$ of $\frac{3}{5}$ of N? $\frac{3}{5}$ of $\frac{1}{4}$ of N?

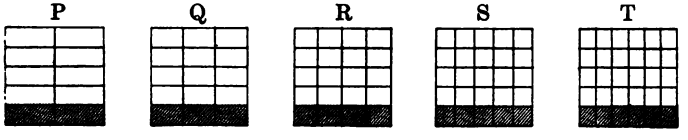
5. What part of O is $\frac{1}{4}$ of $\frac{1}{6}$ of O? $\frac{1}{6}$ of $\frac{1}{4}$ of O? $\frac{1}{4}$ of $\frac{5}{6}$ of O? $\frac{5}{6}$ of $\frac{1}{4}$ of O?

6. Show from K, L, M, N, and O that—

$\frac{1}{4} = \frac{2}{8}$ $\frac{1}{4} = \frac{3}{12}$ $\frac{1}{4} = \frac{4}{16}$ $\frac{1}{4} = \frac{5}{20}$ $\frac{1}{4} = \frac{6}{24}$

$\frac{1}{2} = \frac{4}{8}$ $\frac{1}{2} = \frac{6}{12}$ $\frac{1}{2} = \frac{8}{16}$ $\frac{1}{2} = \frac{10}{20}$ $\frac{1}{2} = \frac{12}{24}$

$\frac{3}{4} = \frac{6}{8}$ $\frac{3}{4} = \frac{9}{12}$ $\frac{3}{4} = \frac{12}{16}$ $\frac{3}{4} = \frac{15}{20}$ $\frac{3}{4} = \frac{18}{24}$



7. What part of P is $\frac{1}{3}$ of $\frac{1}{2}$ of P? $\frac{1}{2}$ of $\frac{1}{3}$ of P?

8. What part of Q is $\frac{1}{3}$ of $\frac{1}{3}$ of Q? $\frac{1}{3}$ of $\frac{1}{5}$ of Q?

9. What part of R is $\frac{1}{5}$ of $\frac{1}{4}$ of R? $\frac{1}{4}$ of $\frac{1}{5}$ of R?

10. What part of S is $\frac{1}{5}$ of $\frac{1}{5}$ of S?

11. What part of T is $\frac{1}{5}$ of $\frac{1}{6}$ of T? $\frac{1}{6}$ of $\frac{1}{5}$ of T?

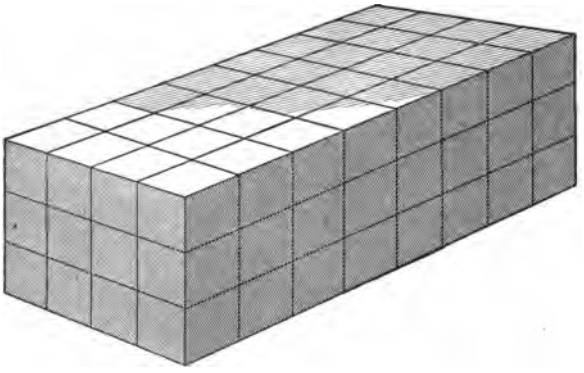
12. Show from P, Q, R, S, and T that—

$\frac{1}{5} = \frac{2}{10}$ $\frac{1}{5} = \frac{3}{15}$ $\frac{1}{5} = \frac{4}{20}$ $\frac{1}{5} = \frac{5}{25}$ $\frac{1}{5} = \frac{6}{30}$

$\frac{2}{5} = \frac{4}{10}$ $\frac{2}{5} = \frac{6}{15}$ $\frac{2}{5} = \frac{8}{20}$ $\frac{2}{5} = \frac{10}{25}$ $\frac{2}{5} = \frac{12}{30}$

$\frac{3}{5} = \frac{6}{10}$ $\frac{3}{5} = \frac{9}{15}$ $\frac{3}{5} = \frac{12}{20}$ $\frac{3}{5} = \frac{15}{25}$ $\frac{3}{5} = \frac{18}{30}$

$\frac{4}{5} = \frac{8}{10}$ $\frac{4}{5} = \frac{12}{15}$ $\frac{4}{5} = \frac{16}{20}$ $\frac{4}{5} = \frac{20}{25}$ $\frac{4}{5} = \frac{24}{30}$



1. In the lower layer of cubic blocks in this solid, how many rows of 4 blocks each? How many blocks in the layer? How is this found? In all 3 layers, or the entire solid, how many blocks are there? How is this found? How then is the number of cubic units in any square-cornered solid found?

2. How many inch cubes are there in a block 2 inches long, 2 inches wide, and 1 inch high? How many inch cubes are there in a block 4 inches long, 2 inches wide, and 1 inch high?

3. How many inch cubes are there in a block 6 inches long, 4 inches wide, and 1 inch high? How many inch cubes are there in one row? How many of these rows are there in the block?

4. How many cubic inches are there in a box that is 4 inches long, 3 inches wide, and 1 inch high? 2 inches high? 3 inches high?

5. How many cubic inches are there in a box 4 inches long, 2 inches wide, and 2 inches high?

6. A block is 5 inches long, 2 inches wide, and 3 inches high. How many cubic inches are there in it?

1. A pencil box containing 24 cubic inches is 3 inches wide and 1 inch high. How long is it?

2. A block is 3 feet long, 3 feet wide, and 3 feet high. How many cubic feet does it contain? What is the area of one side of such a block in square feet? In square yards? How many cubic yards in a block 1 yard long, 1 yard wide, and 1 yard high? How many cubic feet in such a block?

3. A room is 5 yards wide, 7 yards long, and 4 yards high. How many cubic yards are there in the room?

4. A cellar is 7 yards long, 6 yards wide, and 3 yards deep. How many cubic yards of earth were taken out in digging the cellar?

5. The foundation wall of one side of a building is 65 feet long, 4 feet high and $1\frac{1}{2}$ feet wide. How many cubic feet does it contain?

6. A box is 7 feet long, 3 feet wide, and contains 63 cubic feet. How high is it?

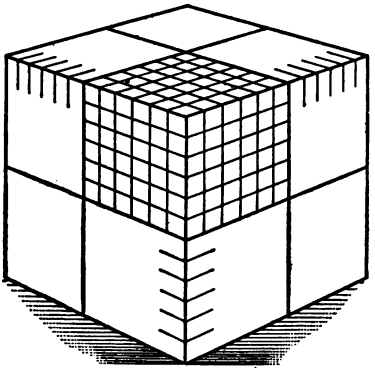
7. A bin is 3 feet wide, 4 feet high, and contains 72 cubic feet. How long is it?

8. A coal-bin is 12 feet long, 6 feet wide, and 7 feet high. How many cubic feet of coal will it hold?

9. A car is 40 feet long, $8\frac{1}{2}$ feet wide, and 8 feet high. How many cubic yards does it contain?

10. A man has 4 bins, each 5 feet long, 4 feet wide, and 3 feet high. How many cubic feet of coal will they all hold?

11. One box is 4 feet long, 3 feet wide, and 2 feet high. A second is 5 feet long, 3 feet wide, and 2 feet high. A third is 6 feet long, 5 feet wide, and 4 feet high. How many cubic feet are there in the 3 boxes?



1. How many edges do you see on this block? How many edges has it?

2. If the edges were all 1 inch long, what would such a block be called?

3. If 12 such cubes are placed in a row, how long is the row?

4. If 12 of these rows are placed side by side, how wide is the whole? How many cubes are used?

5. What figure would the upper surface of the cubes form?

6. How many square inches would there be in this surface? How many inch-cubes would you use in forming this layer?

7. If you should place another layer of cubes on those already used, how many cubic inches would you have? How many, if you used 3 layers? 4 layers? 5 layers? 6 layers? 7 layers?

8. The picture represents 12 layers of inch-cubes. Each layer contains 12 rows of 12 cubes each. How long is each edge? What would you call such a cube? How many cubic inches would it contain?

9. If you should divide one-half of this cube into 4 equal cubes, what would each cube measure?

10. What part of the whole cube would each be?

11. How many cubic inches are there in each part?

12. How many cubic inches in three-eighths of a cubic foot? In seven-eighths? In one-fourth?

1. A berry box is 6 inches long, 4 inches wide, and 4 inches deep. How many such boxes can be packed in a case 2 ft. long, 1 ft. wide, and $\frac{2}{3}$ of a foot high?

2. A gallon contains 231 cubic inches. How many gallons can be put into a pail holding 693 cubic inches?

3. How many boxes 12 inches long, 6 inches wide, and 3 inches high can be packed in a case 6 feet long, 4 feet wide, and 4 feet high?

4. How many cubic feet of air will a glass tank hold, that is 24 inches long, 18 inches wide, and 12 inches high?

5. From a vessel holding 2 cubic feet of water 864 cubic inches were taken. How many cubic inches remain? How many cubic feet?

6. There are 864 cubic inches of liquid in one tank; 2592 cubic inches in another. How many cubic feet in a third tank, holding as much as both together?

7. A man put sand 12 in. deep into a box 9 ft. long and 5 ft. wide. How many cu. ft. of sand were in the box?

8. A wagon box, 3 feet wide and 9 feet long, is $1\frac{1}{3}$ feet deep. How many cubic feet does it hold?

9. A ditch 45 feet long and 2 feet wide contains 630 cubic feet. How deep is the ditch?

10. A freight car is 32 feet long and 6 feet wide inside and is filled with sand $3\frac{3}{4}$ feet deep. How many cubic feet of sand are in the car?

11. A wall is 44 feet long, $5\frac{1}{2}$ feet high, and 18 inches thick. How many cubic feet in the wall?

12. A sidewalk is 6 inches thick and 6 feet wide. How many cubic feet in 124 feet of the sidewalk?

13. In a building there are 18 pillars, 2 ft. by 18 in. and 14 ft. high. How many cubic feet in these pillars?

TABLE OF CUBIC MEASURE

1728	cu. in. (cubic inches)	= 1 cu. ft. (cubic foot).
27	cu. ft. (cubit feet)	= 1 cu. yd. (cubic yard).
128	cu. ft.	= 1 cord (cd.)
231	cu. in.	= 1 gal.
2150½	cu. in. (nearly)	= 1 bu.

1. Add:

7 cu. yd.	6 cu. ft.	27 cu. in.
2	10	250
4	2	41

6 cu. yd.	14 cu. ft.	55 cu. in.
2	2	17
3	3	40

2. Add:

8 cu. yd.	5 cu. ft.	27 cu. in.
3	4	144
20	10	75

1 cu. yd.	5 cu. ft.	160 cu. in.
8	14	750
4	7	818

3. Add:

7 cu. yd.	5 cu ft.	150 cu. in.
5	10	364
8	14	225
9	3	600

7 cu. yd.	26 cu. ft.	600 cu. in.
3	24	420
2	12	708
9	17	800

4. Subtract:

10 cu. yd.	25 cu. ft.	120 cu. in.
8	16	90

9 cu. yd.	18 cu. ft.	350 cu. in.
5	9	275

5. Subtract:

[illegible] yd.	20 cu. ft.	8[illegible]
[illegible]	13	[illegible]

11 cu. yd.	15 cu. ft.	920 cu. in.
6	7	256

10. [illegible]

11. [illegible]

12. [illegible]

foot? [illegible]

12 cu. yd.	6 cu. ft.	1296 cu. in.
		4

1. A boy cut enough wood to make a pile 4 ft. long, 2 ft. wide, and 2 ft. high. What part of a cord did he cut?

2. What must be the cubic contents of a jar to hold $\frac{2}{3}$ of a gallon? $2\frac{1}{3}$ gal.? $4\frac{4}{7}$ gallons?

3. How many cubic inches are there in a box holding 2 bu.? $\frac{1}{2}$ bu.? $5\frac{1}{3}$ bu.? $4\frac{1}{8}$ bu.? $6\frac{1}{4}$ bushels?

4. A cord of wood is usually piled 8 ft. long and 4 ft. wide. How high is it?

5. A box holds 16 bu. How many cubic inches does it contain?

6. How many cu. yd. of earth must be dug out to make a cellar, 24 ft. long, 21 ft. wide, and 12 ft. deep?

7. In making a cellar 36 ft. long and 18 ft. wide, 6804 cu. ft. of earth were taken. How deep was the cellar?

8. How many cu. yd. of earth were taken from a tunnel that is $9\frac{1}{2}$ ft. high, 80 ft. long, and $12\frac{1}{2}$ ft. wide?

9. A cubic foot of water weighs 1000 ounces. What is the weight in pounds of water enough to fill a trough 6 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. deep?

10. Oil weighs $\frac{4}{5}$ as much as water. What is the weight in pounds of a cubic foot of oil?

11. A street sprinkler holds 168 cu. ft. of water. How many pounds does it hold?

12. Such a street sprinkler is emptied every 24 minutes during 9 hours. How many buckets of water are used if a bucket holds $\frac{1}{3}$ of a cubic foot?

A rectangular tank, 6 ft. wide, 10 ft. long, and 3 ft. is full of water. What is the weight of the water?

35 cu. ft. of coal weighs about 2000 pounds. How tons does a wagon box 7 ft. long, 3 ft. wide, and high hold when loaded full?

1. Add, prove, and note the time you take:

3	4	6	9	6	6	8	9	4	5	5	5
3	2	3	1	3	7	6	6	5	4	6	5
3	6	2	3	4	2	1	8	6	6	7	7
2	7	4	2	5	1	4	1	7	2	8	7
5	5	7	6	2	3	2	2	8	3	9	6

2. Add, prove, and note the time you take:

2	1	2	2	4	5	9	7	7	8	9	7
2	2	3	4	3	8	8	6	7	7	8	3
2	3	4	6	1	2	3	2	5	6	7	8
3	4	5	3	2	3	2	3	4	5	6	2
7	5	2	8	8	6	1	5	3	4	5	6

3. Subtract, and note the time you take:

268	321	463	866	711	301	400	600	700
129	148	375	688	327	199	298	378	509

4. Multiply, prove, and note the time you take:

28	68	75	68	47	43	59	68	75	86
32	42	37	18	26	47	28	55	56	38

5. Divide, and note time taken:

$4\overline{)176}$	$6\overline{)738}$	$6\overline{)1386}$	$8\overline{)1880}$	$9\overline{)3249}$
$21\overline{)4242}$	$23\overline{)2553}$	$30\overline{)1620}$	$40\overline{)2400}$	$50\overline{)2700}$

6. Prove the results of problem 3, by adding the *difference* to the subtrahend in each case and comparing the sum with the minuend. Note time.

7. Prove the results of 5 by multiplying quotient by divisor and comparing the product with the dividend.

TABLE OF LIQUID MEASURE

4	gi. (gills)	= 1 pt. (pint).
2	pt. (pints)	= 1 qt, (quart).
4	qt. (quarts)	= 1 gal. (gallon).
31½	gal. (gallons)	= 1 bbl. (barrel).
231	cu. in.	= 1 gal.

1. Add:

4 gal. 2 qt. 1 pt.	7 gal. 3 qt.	2 bbl. 14 gal. 1 qt.
3 1 1	2 2 pt.	3 6 3
1 bbl. 20 gal. 1 qt.	15 gal. 1 pt.	16 gal. 1 qt.
11 1	2 3 qt.	1 bbl. 15 1

2. Subtract:

7 gal. 3 qt. 2 pt.	3 bbl. 30 gal. 2 qt.	1 bbl. 31 gal. 2 qt.
3 2 1	1 21	8 1

3. Multiply:

2 gal. 3 qt. 1 pt.	7 gal. 1 qt. 2 pt.	2 bbl. 10 gal. 2 qt.
2	4	3

4. A milkman starts with 42 gal. of milk. He sells $\frac{2}{3}$ of it to private customers and the rest to a hotel. How many quarts does he sell to the hotel?

5. A druggist put 1 qt. of liquid into bottles holding $\frac{1}{2}$ gi. each. How many bottles did he use?

6. How many jelly glasses holding $\frac{2}{3}$ of a pt. each can be filled from 1 gal. of jelly?

7. How many cubic inches are there in 32 liquid quarts? How many more cubic inches are there in 32 dry quarts (1 dry qt. = $\frac{1}{32}$ bu.) than there are in 32 liquid quarts?

1. How many pint bottles will hold 2 gal. 1 pt. of vinegar? What is it worth at 13¢ a quart?

2. If a gallon of olive oil cost $4, what will 5 pt. cost?

3. From a milk can holding 8 gal., $\frac{3}{16}$ of the milk was spilled. How many quarts were left? How many gallons?

4. How much ice cream will a man make if he uses a gallon and a half freezer and makes it full twice, and half full the third time?

5. How many gal. in 412 gills?

6. What part of 12 gal. is $4\frac{1}{2}$ gallons?

7. A man sold 12 cans of mineral water, each holding 6 gal. at 15¢ per gal. How much did he receive?

8. How many oil barrels, each holding 45 gal., can be filled from a tank holding 10,800 gal. of oil?

9. A hotel uses 25 gal. 3 pt. of milk each day. How much does it use in 3 weeks?

10. There are 231 cu. in. in 1 gal. How many cu. in. in a bottle holding 2 quarts?

11. How many cubic inches in a cistern holding 10 bbl.? 12 bbl.? 16 barrels?

12. From a barrel of vinegar a grocer fills 2 four-gallon jugs and puts 3 gal. and 1 qt. in another jug. How many gal. had he left in the barrel?

13. A man sells 3 gal. and 2 qt. of molasses to one customer, 2 gal. and 1 qt. to another customer, and 1 qt. and 1 pt. to a third. How many quarts did he sell in all? How many gallons?

14. From a barrel full of rain water, 80 qt. were taken out at different times. The water remaining in the barrel measured 40 qt., the rest having evaporated. How much water evaporated?

1. A brick mason agrees to build a chimney for \$72. It takes 10 days to do the work and he pays each of his 2 helpers \$1.50 a day. What is his own pay per day?

2. A man agreed to haul away 1560 cu. yd. of clay for \$264. He paid 4 teamsters \$3.90 each per day for 13 days. How much did he have left?

3. At the rate of 13¢ per cu. yd., how many yards did each teamster haul to earn what he received?

4. A man hauls sand for 9¢ per cu. yd. If his wagon holds $1\frac{1}{2}$ cu. yd. and he hauls 18 loads per day, what is his daily pay?

5. A newsboy buys his papers at the rate of 10 for 6¢ and sells them for 1¢ each. How much will he gain if he sells 75 papers? 120 papers?

6. He sells 45 on Monday, 54 on Tuesday, 81 on Wednesday, and 70 on Thursday. What does he gain in the 4 days?

7. On Friday he buys 100 papers and sells all but 5 that are spoiled by the rain. What does he receive for his work on Friday?

8. A newspaper prints $1\frac{1}{2}$ million copies in 6 days. At the end of the week 13,526 copies had been given away and 29,674 copies were left on hand. What was the average daily paid circulation?

9. If $\frac{1}{2}$ of these papers are sold by newsboys, how many newsboys must there be, if each one sells 100 papers every day?

10. A man divides 80 acres of land into streets and building lots. The streets take up $\frac{1}{16}$ of the land, and the remainder is divided into blocks each containing 3 A. How many blocks are there?

TABLE OF DRY MEASURE

2 pt. (pints) = 1 qt. (quart).
8 qt. (quarts) = 1 pk. (peck).
4 pk. (pecks) = 1 bu. (bushel).

1. Add:

1 pk. 6 qt.	1 bu. 2 pk. 3 qt.	2 bu. 3 pk. 4 qt.
2 1	2 1 5	3 4
2 pk. 5 qt.	5 bu. 2 qt.	3 bu. 3 pk. 7 qt.
2	3 pk. 6	1

2. Subtract:

3 pk. 6 qt.	3 bu. 2 pk. 7 qt.	6 bu. 1 pk. 5 qt.
2 4	2 5	1 1 1
2 pk. 4 qt.	8 bu. 3 pk. 5 qt.	12 bu. 2 pk. 6 qt.
1 4	2 2 2	7 4

3. Multiply:

2 pk. 3 qt.	3 bu. 1 pk., 3 qt.	4 bu., 1 pk. 1 qt.
2	3	4

4. How many bushels in 128 pk.? In 150 pecks?

5. How many quart boxes will 1 bu. 3 pk. 2 qt. fill?

6. Find cost of 3 pk., 6 qt., 1 pt. of nuts, at $12\frac{1}{2}¢$ a pint.

7. In 96 qt. how many pecks? How many bushels?

8. What part of 7 bu. is 7 pk.? 7 quarts?

9. How many quarts of cherries can be bought for $2, if 1 bushel of cherries is worth $3.20?

10. A teamster feeds his horses 36 qt. of oats a day. How long will 120 bu. of oats last him? What does it cost him a day when oats are worth 27¢ a bushel?

1. From a sack holding 3 bu. of peanuts, 25 qt. were taken. How many bushels remained?

2. A bushel and a half of plums was divided equally among 12 people. How many quarts did each receive?

3. How many pecks of beans does a man sell, if he sells 3 qt. to each of 8 customers?

4. A woman measuring out $2\frac{1}{2}$ qt. of flour uses a pint measure. How many times does she fill the measure?

5. A barrel of apples was sold in 3 lots. The first sale was 1 bu. and 2 pk.; the second 2 pk.; the third $1\frac{1}{2}$ bu. How many bushels were there in the barrel?

6. From 3 bushels of peas, a man sells $\frac{1}{2}$ peck to one customer, 4 qt. to another, 1 pk. to another. How much has he left?

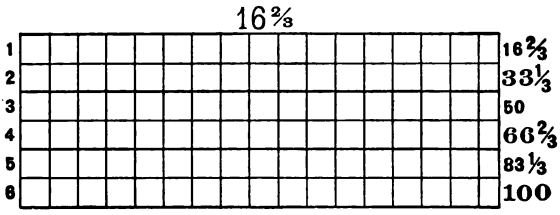
7. If $\frac{1}{2}$ peck of peaches when canned make 3 qt., how many bu. must be bought to make 36 qt. when canned?

8. A man bought at a feed store 5 bu. of corn, 2 bu. and 3 pk. of oats, and $1\frac{1}{2}$ bu. of mixed feed. He had at home 3 pk. of corn, $\frac{1}{2}$ bu. of oats and 1 pk. of ground feed. How many bushels of feed did he have after receiving what he bought?

9. A farmer used for food $50\frac{1}{4}$ bu. of potatoes. He saved 9 bu. 3 pk. for planting and sold $117\frac{1}{2}$ bushels. How many bushels did he raise that year?

10. 240 boxes of peaches, holding 1 pk. each, were shipped to market. The fruit was picked in one-half bushel baskets. How many baskets of fruit were there?

11. A fruit dealer bought 2 crates of berries, each holding 24 qt., and 6 crates, each holding 32 qt. He put the berries into pint boxes and sold them at 10¢ a box. How much money did he receive?



1. Add:

$16\frac{2}{3}$	$16\frac{2}{3}$	$16\frac{2}{3}$	$33\frac{1}{3}$	$66\frac{2}{3}$	$83\frac{1}{3}$
$16\frac{2}{3}$	$33\frac{1}{3}$	50	$33\frac{1}{3}$	$16\frac{2}{3}$	$16\frac{2}{3}$

2. $16\frac{2}{3} \times 3 = ?$ $16\frac{2}{3} \times 6 = ?$

$16\frac{2}{3} \times 2 = ?$ $16\frac{2}{3} \times 5 = ?$

$16\frac{2}{3} \times 4 = ?$ $16\frac{2}{3} \times 9 = ?$

3. $16\frac{2}{3}$ is what part of 50? $66\frac{2}{3}$? $33\frac{1}{3}$? $83\frac{1}{3}$?

4. $33\frac{1}{3}$ is what part of 50? $66\frac{2}{3}$? $83\frac{1}{3}$? 100?

5. 50 is how many $16\frac{2}{3}$'s? $33\frac{1}{3}$'s?

6. $66\frac{2}{3}$ are how many $16\frac{2}{3}$'s? $33\frac{1}{3}$'s? 50's?

7. $83\frac{1}{3}$ are how many $16\frac{2}{3}$'s? $33\frac{1}{3}$'s? 50's? $66\frac{2}{3}$'s?

8. 100 is how many $16\frac{2}{3}$'s? $33\frac{1}{3}$'s? 50's? $66\frac{2}{3}$'s? $83\frac{1}{3}$'s?

9. A man paid $\frac{1}{6}$ of a dollar each for 15 books. How much did they cost him?

10. A boy $33\frac{1}{3}$ miles from home, rode $\frac{1}{2}$ the distance on his wheel. How many miles did he ride?

11. A piece of cloth 3 yd. long sold for 50¢. What was the price per yard?

12. A girl picked 100 qt. of berries in 6 days. What was the average amount per day?

13. A hundredweight of grain was divided into 3 equal amounts? How many pounds in each?

14. From a tank of water, $16\frac{2}{3}$ gal. were drawn out, which was $\frac{1}{6}$ of the amount remaining. How many gallons were left in the tank?

1. A schooner brought 48,972 spruce trees from northern Michigan to Chicago at Christmas time. How much were they worth at 90¢ a dozen?

2. A blacksmith shoes 42 horses at $2 each in 1 week. The shoes and nails cost him $8, shop rent $12, and he pays each of his two helpers $15 per week. What was his share of the money received?

3. A teamster has his two horses shod all around, twice each month, during Dec., Jan., and Feb., and once a month during the remainder of the year. How much does it cost him a year at $2 per shoeing for each horse?

4. A cooper made 1000 butter tubs at 12¢ each. If he paid $\frac{1}{6}$ of the price for lumber and $\frac{1}{2}$ of it to his workmen, how much did he have left?

5. Each tub holds 24 pounds. What is 1000 tubs of butter worth at 21¢ a pound?

6. There are 630 sq. yd. of lathing needed in a new house and it can be done in 6 days. Will a man earn more by doing it by the day at $3 per day or by the square yard at 3¢ per square yard?

7. If 54 laths cover 4 sq. yd., how many will be needed for 600 sq. yd.? How many bundles of 50 laths each?

8. Each lath requires 6 nails, and 54 laths cover 4 sq. yd., how many nails will 600 sq. yd. use? How many pounds, allowing 400 nails to the pound?

9. A carpenter works $5\frac{1}{2}$ days in a week at $3 per day, but breaks a saw worth $1.40 and loses a hammer worth 90¢. What is the week's work worth to him?

10. A painter has 3 helpers at $3.50 each per day and 2 helpers at $3 each. What must he charge for a week's work to have $25 for himself?

For convenience in reading, numbers are separated by commas into groups of 3 figures, beginning at ones. The 3-figure groups are called *periods*, thus:

Thousands				Units		
Hundreds.	Tens.	Ones.		Hundreds.	Tens.	Ones.
		6	,	1	2	3
	4	9	,	8	5	2
7	9	5	,	6	4	8

In reading these numbers the figures of each period are read as though they stood alone and then the name of the period is added, thus:

6 thousand 123;
49 thousand 852;
795 thousand 648.

1. Read the following numbers:

8,000	64,387	910,643
7,004	30,300	606,606
4,345	82,082	450,450
5,060	47,740	745,547
3,400	53,961	300,300

2. The number 2,461,375 shows the beginning of a new period, called *millions*. This number is read:

2 million 461 thousand 375.

Read the following numbers:

7,000,000	5,406,406
82,000,000	47,340,340
436,000,000	202,202,202

The method, just described, of representing numbers by figures is called the Arabic notation.

There is another method of writing numbers, in which letters are used. It is called the Roman notation.

1. Write the Arabic figure for each of the following:

I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XX.

L = 50, C = 100, D = 500 and M = 1000.

If a letter is followed by one or more letters of equal or less value, the *sum* of all is the value of the number represented. Thus: VI = 6; XI = 11; XX = 20; CLX = 160; DC = 600.

If a letter is followed by another of greater value, the *difference* of the two is the value of the number represented. Thus: IV = 4; IX = 9; XIX = 19; XL = 40; CD = 400.

2. Give the value in the Arabic notation of the following numbers:

XIV	DIX	XIV	XXVII	CCCXVI
XIX	LCI	XCXI	XXXIX	MCLXIX
LIV	XLIV	LXIX	CLXXI	MCLVI
CXI	XLIX	XXXIV	LXXVI	MCMVIII

Write the following numbers in the Roman notation: 9, 4, 6, 14, 29, 13, 78, 44, 83, 59, 94, 96, 104, 199, 335, 549, 2000, 1908.

Write answers to the following in Roman numerals:

3. Columbus discovered America in MCDXCII. 20 years later Florida was explored. In what year was Florida explored?

4. The first battle of the Revolution was fought in MDCCLXXV. The last battle was fought 6 years later. What was the year of the last battle?

5. Washington was elected President in MDCCLXXXIX. Roosevelt was elected 115 years afterward. In what year was Roosevelt elected?

1. Add:

(1)	(2)	(3)	(4)	(5)	(6)
7843	32765	25987	96875	81818	247
8789	89247	6586	40984	92193	91838
9576	76348	78379	50839	87689	9705
2589	20873	96468	9787	76434	87278
8956	94608	980	67898	68979	3849
3210	13495	20876	76580	37590	89878
7029	68950	67099	54777	89763	79929
47992					
334					

In long columns the number to be carried may be indicated by writing it underneath the column as in problem (1).

To prove the work, add from the top, downward.

2. Add the following problems in the usual way:

(1)	(2)	(3)	(4)
68	238	237	24567
97	472	48984	89012
89	836	3789	4567
43....297	980.....?	54976.....?	89876.....?
30	722	500	54378
78	146	3897	98989
83	348	57878	24864
29....220	765.....?	36498.....?	3099.....?
48	897	9889	87655
67	305	65847	98788
98	969	678	4890
88....301	878.....?	88769.....?	86767.....?
818 818			

Another method of proof is to think the columns divided into parts, add the parts, then add these partial sums.

1. 6384 *The Minuend is the number subtracted from.*

1945 *The Subtrahend is the number subtracted.*

4439 *The Difference, or Remainder is the result.*

6384 The sum of the difference and subtrahend should be the minuend. If so, the work is correct.

2. Subtract the following and prove by adding:

(1)	(2)	(3)	(4)	(5)	(6)
8346	24890	36745	48234	57855	72180
5838	17901	17829	29018	29666	23092

(7)	(8)	(9)	(10)	(11)	(12)
62387	83475	90281	38297	27666	87726
34299	56077	37345	19088	18785	68640

(13)	(14)	(15)	(16)	(17)	(18)
418967	687240	723485	868240	927200	707241
229875	478119	438907	372906	418117	354438

(19)	(20)	(21)	(22)	(23)	(24)
381487	592173	600840	821380	727248	917288
191598	394205	236450	291653	570649	129399

3. A man began business with $5275.75; in five years he had $22,794.50. How much had he gained?

4. One road is 20 mi. 160 rd. long; another is 14 mi. 80 rd. long; how much longer is the first than the second?

5. A cotton dealer bought 328,900 lb. of cotton one year, and 715,600 lb. the next. How many pounds more did he buy the second year than the first?

6. One vessel is valued at $1,250,000; another at $975,800. What is the difference in value?

1. Multiply 892 by 235. Which number is the multiplicand? The multiplier? The product?

To how many times 892 is the sum of 5 times 892 + 30 times 892 + 200 times 892 equal?

FULL FORM			CONVENIENT FORM
Multiply 892			892
by 235			235
4460	=	5 × 892	4460
26760	=	30 × 892	2676
178400	=	200 × 892	1784
209620	=	235 × 892	209620

2. Multiply the following:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
563	589	789	582	678	765	899
365	589	579	376	497	689	655

3. Multiply (1) 729 by 460; (2) 476 by 308.

(1)	(2)
729	476
460	308
43740	3808
2916	1428

4. Compare 4×5 with 5×4. Compare 892×235 with 235×892. In multiplication why do we commonly use the smaller number for the multiplier? Give a way of proving that a product is correct.

5. A steamer burns 378 lb. of coal in going 1 knot. How many pounds will it burn in going 15,288 knots?

6. A man's daily income is \$3.65. How much is his yearly income?

1. Find the products of the following, and prove results:

573×248	384×537	735×376	487×789
858×305	275×937	972×219	976×253
835×583	508×607	506×429	4005×129
968×675	651×234	309×150	2060×456
809×584	943×923	847×907	3070×250
548×394	475×406	653×295	1022×284

2. An elevator in a tall building goes up 165 ft. and down the same distance each trip. How many feet does it go in 152 trips?

3. It costs 56¢ a word for a cablegram from Japan to Chicago. How much will a message of 1213 words cost?

4. There are 40 street lamps on 1 mile of street, each burning 18 cu. ft. of gas every night. How much gas will they burn in the month of April?

5. A library has in one case 276 books, which contain, on an average, 304 pages each. How many pages of reading matter in the bookcase?

6. There are 897 hills of corn in a single row and 364 such rows in a field. How many hills of corn in the entire field?

7. How many square feet in an acre? If 189 pounds of water fall on one square foot in a year, how many pounds will fall on an acre in a year?

8. There are 12 elevators in a building. Each makes 94 round trips in a day and carries, on an average, 4 passengers each way. How many passengers make single trips on all of them in a day?

9. In a single layer in a wall there are 964 bricks. The wall is 197 bricks high. How many bricks does it contain?

1. Divide 21,816 by 72.

$$72\overline{)21816}\quad 303$$

216

216

216

What does 303×72 equal? How, then, may the correctness of the work of division be proved?

$21{,}816 \div 72 = 303.$

2. Divide 15,250 by 61.

$$61\overline{)15250}\quad 250$$

122

305

305

3. Divide 5130 by 342.

$$342\overline{)5130}\quad 15$$

342

1710

1710

4. Divide 25,215 by 105.

$$105\overline{)25215}\quad 240\tfrac{15}{105}$$

210

421

420

15 remainder.

Proof: 105 divisor.

240 quotient.

4200

210

25200

15 remainder.

25,215 dividend.

To test the correctness of division, multiply the divisor by the quotient, and add the remainder if there is one. The result should equal the dividend.

5. Divide and test:

$3767 \div 38$	$99{,}684 \div 234$
$7873 \div 41$	$91{,}464 \div 111$
$7692 \div 32$	$94{,}770 \div 135$
$67{,}654 \div 53$	$95{,}928 \div 806$
$26{,}754 \div 64$	$90{,}750 \div 125$
$95{,}637 \div 75$	$68{,}331 \div 911$

1. There were 23 cars in a train and 21 cattle in each car. The entire weight of the cattle was 645,502. What was the average weight per head?

2. An apartment building containing 48 apartments cost $132,240. What was the average cost of an apartment?

3. A farmer paid $20,000 for 324 acres of land. How much did it cost him per acre?

4. Solve the following exercises:

$516\overline{)220332}$	$718\overline{)92622}$	$356\overline{)182984}$	$729\overline{)91125}$
$618\overline{)400276}$	$317\overline{)687020}$	$540\overline{)72689}$	$720\overline{)840671}$
$427\overline{)600000}$	$825\overline{)872640}$	$207\overline{)907060}$	$421\overline{)117000}$
$5027\overline{)702060}$	$1702\overline{)347020}$	$1202\overline{)987261}$	$4800\overline{)960000}$
$5200\overline{)987654}$	$3702\overline{)587869}$	$1234\overline{)765432}$	$1067\overline{)987000}$

5. On a certain trip the traveling expenses for one person to London and return were $325. How many such trips would $9100 furnish?

6. A proofreader counted the number of words in a book, finding 88,440 words on 268 pages. How many words were there on an average page?

7. A wealthy man gave to the 4 fire departments of a village (each department employing 27 men) $32,400 as a donation, to be divided equally among the firemen. How much did each man receive?

8. A woman donated $1701 to buy books for the school children of a certain town. There were 567 school children. How much was the average donation per pupil?

1. A lumber vessel carried 887,392 barrel staves. Allowing 16 staves to the barrel, how many barrels can be made from them?

2. A ship traveled about 15,228 nautical miles in 66 days, stopping 12 days on the way for coal and other supplies. Find the average speed per hour.

3. The Danube river is 1725 miles long; the Rhine 600, and the Rhone 580. How many times as long as the Thames, which is 220 miles long, is their united length?

4. There are 102 counties in Illinois and the area of the state is 56,650 square miles. What is the average size of a county?

5. Chicago's area is 119,689 acres. How many square miles of land in Chicago?

6. A field of corn has 229,559 hills. There are 291 equal rows. How many hills in each row?

7. A rectangular farm contains 552,866 square yards. One end is 563 yd. long. How long is one side?

8. A rectangular field contains 926,156 square feet. One end is 679 feet long. How long is one side?

9. At a brick yard 3,276,224 bricks were made during the season. They were hauled away in 824 equal loads. How many bricks were hauled at a load?

10. $48,077.29 was divided equally among 709 men. How much did each man receive?

11. A merchant sold 999 bicycles for $41,793.72. What was the average selling price of a bicycle?

12. In four years a factory uses 1,434,160 pounds of coal. What was the average amount used per week?

13. The Atlantic Ocean, in the deepest part, is 27,366 feet deep. What is the depth in miles?

1. Add, and note time taken:

7	8	9	9	28	44	25	75	4	6
1	2	8	9	31	16	64	1	17	604
6	4	7	7	46	21	71	325	6	28
3	5	6	2	58	17	29	26	297	9
7	1	5	4	30	64	86	2	8	190

2. Subtract, and note time taken:

8402	7008	8002	6006	7010	20104	30607
1384	3261	1247	2987	3265	8638	29838

3. Multiply and note time taken:

124	216	328	560	762	408	902	988
32	36	43	44	48	57	86	97
328	482	614	736	808	588	961	998
216	315	402	207	304	608	808	786

4. Divide, and note time taken:

$12\overline{)232}$	$21\overline{)294}$	$18\overline{)576}$	$27\overline{)351}$
$33\overline{)1254}$	$13\overline{)806}$	$24\overline{)1008}$	$43\overline{)1806}$

5. Divide, and note time taken:

$36\overline{)4428}$	$42\overline{)13692}$	$54\overline{)22734}$	$68\overline{)30736}$
$86\overline{)77572}$	$97\overline{)95836}$	$138\overline{)45264}$	$216\overline{)70848}$

6. Add, and note time taken:

4308	8063	89	7	70006
209	214	1097	1838	6318
98	9098	7809	2785	87292
1097	935	364	8190	2364
672	1710	2079	706	555

TABLE OF WEIGHT

16 oz. (ounces) = 1 lb. (pound).
100 lb. (pounds) = 1 cwt. (hundredweight).
20 cwt. or 2000 lb. = 1 T. (ton).

1. How many ounces in 8 lb.? In 14 lb.? In 20 lb.?

2. How many lb. in $3\frac{1}{2}$ cwt.? In $7\frac{1}{5}$ cwt.? In $9\frac{1}{4}$ cwt.?

3. How many lb. in 1 T.? In 4 T.? In $3\frac{1}{4}$ T.?

4. How many oz. in $\frac{1}{2}$ cwt.? In $2\frac{1}{4}$ cwt.? In $3\frac{2}{5}$ cwt.?

5. How many oz. in $\frac{1}{2}$ T.? In $3\frac{1}{5}$ T.? In $2\frac{1}{8}$ T.?

6. How many cwt. in $\frac{1}{2}$ T. and in $\frac{1}{4}$ T. together?

7. Add:

2 T.	17 cwt.	25 lb.
1	2	75

1 T.	8 cwt.	60 lb.
4	1	40

10 cwt.	25 lb.	8 oz.
5	30	6
4	44	2

9 cwt.	20 lb.	4 oz.
4	9	9
6	18	2

8. Subtract:

2 T.	15 cwt.	14 lb.
2	7	8

2 T.	12 cwt.	9 lb.
1	8	6

15 cwt.	75 lb.	8 oz.
8	35	5

16 cwt.	43 lb.	12 oz.
14	23	8

9. Multiply:

3 T.	5 cwt.	15 lb.
		4

2 T.	3 cwt.	20 lb.
		5

2 cwt.	9 lb.	2 oz.
		8

4 cwt.	3 lb.	4 oz.
		4

10. Divide:

2)4 T. 18 cwt. 50 lb.

3)6 T. 15 cwt. 48 lb.

1. At 10¢ an oz. how much will $3\frac{1}{2}$ lb. of cinnamon cost? $4\frac{1}{4}$ pounds?

2. What will a ton of hay cost at $\frac{3}{4}$¢ a pound?

3. A bushel of wheat weighs 60 pounds. How many bushels are there in a load of wheat weighing 1 T. 6 cwt. and 40 pounds?

4. A family uses 75 lb. of ice every day for 20 days. What does it cost them at 35¢ per hundredweight?

5. From 10 cwt. of sugar a grocer sold 8 cwt., 40 lb., 12 ounces. How much was left?

6. What was the cost of 3 lb., 8 oz. of meat at 16¢ a lb. and 2 lb., 12 oz. of butter at 20¢ a pound?

7. A bushel of oats weighs 32 pounds. How many bushels in 80 hundredweight?

8. From a keg of nails weighing 1 cwt., at one time, 15 lb. and 8 oz. were taken; at another, 9 lb. and 12 oz. What was the weight of the nails remaining?

9. How many bales of cotton, each weighing 450 lb. may be shipped on a vessel which can carry 2250 tons? On a vessel that can carry 3600 tons?

10. What will it cost to have 7 tons of coal hauled at 75¢ a load of 3500 pounds?

11. 500 bbl. of flour are shipped in 20 equal loads. How many tons in each load? (1 bbl. weighs 196 pounds.)

12. A man hauled 8 loads of wheat, 35 bu. at a load. A bushel of wheat weighs 60 pounds. What was the weight of the 8 loads? How many hundredweight in each load?

13. A farmer owning 700 bu. of oats sold 9 loads of 2240 pounds each. One bushel weighs 32 pounds. How many bushels had he left?

TABLE OF TIME

60 sec. (seconds) = 1 min. (minute).
60 min. (minutes) = 1 hr. (hour).
24 hr. (hours) = 1 da. (day).
7 da. (days = 1 wk. (week).
365 da.
52 wk. 1 da. } = 1 yr. (year).
12 mo.

1. How many months in a $\frac{1}{2}$ yr.? In $3\frac{1}{4}$ yr.? In $5\frac{1}{6}$ yr.?

2. How many minutes in 3 hr.? In $2\frac{1}{3}$ hr.? In $1\frac{1}{4}$ hr.? In $\frac{3}{4}$ hr.? In one day?

3. In 4 yr. how many days? In $2\frac{1}{5}$ yr.? In $\frac{4}{5}$ yr.?

4. How many seconds in 5 min.? In one hr.? In $\frac{1}{2}$ hr.? In $\frac{1}{2}$ day?

5. Add:

8 hr. 30 min. 15 sec.	3 mo. 4 wk. 7 da.
3 29 45	8 3 12
1 yr. 8 mo. 15 da.	2 yr. 6 mo. 10 da.
3 2 5	2 4 15

6. Subtract:

24 hr. 50 min. 30 sec.	4 yr. 8 mo. 15 da.
8 30 15	1 2 6
6 yr. 4 mo. 20 da.	5 yr. 9 mo. 6 da.
3 3 12	4 8 5

7. Multiply:

4 hr. 15 min. 30 sec.	3 hr. 15 min. 20 sec.
2	3
4 yr. 2 mo. 4 da.	5 yr. 2 mo. 3 da.
5	6

1. Divide:

2)6 hr. 45 min. 15 sec. 6)24 hr. 30 min. 18 sec.

2)4 yr. 8 mo. 12 da. 5)5 yr. 10 mo. 15 da.

2. Find the time from June 6, 1885, to Aug. 12, 1889. Use the numbers that indicate the order of the months in the year instead of the names. Call Jan., 1; Feb., 2, etc.

Aug. 12, 1889 = 1889 yr. 8 mo. 12 da.
June 6, 1885 = 1885 6 6

4 2 6

3. Find the time from:

Jan. 2, 1865, to June 5, 1887;
Mar. 12, 1885, to May 21, 1889;
July 21, 1872, to Sept. 26, 1890;
Feb. 15, 1860, to Oct. 29, 1901;
Nov. 2, 1889, to Dec. 28, 1904;
Apr. 10, 1879, to Aug. 25, 1908.

4. A boy works $\frac{1}{3}$ of each day (24-hr.) for 6 days at 20¢ an hr. What is his week's salary? He works 10 hr. overtime; half of that time at 15¢ a half hour and the rest at 20¢ a half hour. What is his pay for the extra work?

5. A man leaves his office for home at 5:45 P.M., and arrives 45 min. later. What time does he reach home?

6. A fast mail train runs from Chicago to Burlington in 5 hr. 20 min. 45 sec. A freight train runs the same distance in 9 hr. 40 minutes. How much longer does it take the freight train?

7. James was born September 18, 1887, and Willie was born December 21, 1892. How much older is James?

1. There are 18 windows in each side of a car. How many windows are there in the sides of 397 cars?

2. There are 319 pages in a book. How many pages are there in 369 such books?

3. In a 7-story building, there are on each of the sides, 27 windows on each story; there are on each of the ends 16 windows on each story. How many windows in all?

4. A street paved with stone blocks contains 65 stones in width and 786 in length. One man must buy half of the stones. If they cost him 7¢ each, how much does he pay for them?

5. How many grains will 1648 silver dollars weigh, if one dollar weighs 412½ grains?

6. A man rides his bicycle 116 miles a day. How far from the city can he go and return in 12 days?

7. It is 195 feet between telegraph poles. How much wire is needed to put up 6 wires on 257 poles?

8. A merchant bought 97 rolls of carpet, 27 of them containing 56 yards to the roll; 34 containing 59 yards to the roll, and the remainder 63 yards to the roll. How many yards did he buy?

9. A manufacturer sold 65 cases of shoes at $2.75 a pair, each case containing 6 dozen pairs. What was the value of his sale?

10. A farmer had 2365 bushels of wheat. He sold 1240 bushels at 65¢ a bushel; 643 bushels at 85¢ a bushel, and the remainder at 96¢ a bushel. How much did he get for the entire crop of wheat?

11. A merchant's sales were $127 a day for 23 days, $156 a day for 19 days, and $113 a day for 32 days. How much was the total amount of his sales?

1. New York is 91 miles from Philadelphia. A man made 26 round trips from New York to Philadelphia. How many miles did he travel?

2. A man lived 27 miles from Washington. He came into the city and returned 6 days a week for 14 weeks. How many miles did he travel?

3. A manufacturer sold 687 bicycles at $47 apiece. How much did he get for them?

4. An agent sold 163 reapers at $145 each. What was the amount of his sale?

5. A square building is 115 ft. high and 122 ft. wide. What is the area of one of the sides in square feet?

6. The same building is covered with a flat roof. How many square feet in the roof?

7. 53 cars stand in a solid blockade on the street. 26 of them are 32 ft. long and the others are 37 ft. long. How long is the blockade?

8. A street car company sold 540 horses at $62.25 a head. For how much did the horses sell?

9. A man works $8\frac{1}{2}$ hours a day, 6 days in the week, for 26 weeks. How many minutes does he work?

10. For every 25 papers a boy delivered, he received 13¢. If he delivered 175 papers each work day for 2 weeks, how much money did he receive?

11. I burned a quart of oil a week. The oil cost me 13¢ a gallon. How much did the oil cost me for a school year of 40 weeks?

12. An elevator boy received $13 a month as wages. At the end of the year he had spent $19 for clothing, $32 for car fare, $4.50 for books and had given his mother $48. How much money had he left?

A man had one five-dollar bill, two two-dollar bills, three one-dollar bills, two half-dollars, three quarters, four dimes, six nickels, and three pennies. How can he make even change for any one of the following purchases:

1. A bicycle tire at \$2.75?

2. A hat at \$2.65 and a necktie at 50¢?

3. A vest at \$3.50 and a dozen collars at 95¢?

4. One book at \$1.25 and another at 38¢?

5. A box of paper at 50¢, an inkstand at 65¢, and a fountain pen at \$1.95?

6. A pair of shoes at \$2.95?

7. Railroad fare at \$4.42?

8. Hotel bill for $2\frac{1}{2}$ days at \$2.50 a day?

9. A magazine, 35¢, a newspaper, 2¢, and a sheet of paper, envelope, and stamp, 5¢?

10. A bicycle suit at \$6.70? Repairing bicycle, \$4.70?

A boy got at the bank, at different times, the amounts of change indicated below. Find how many of each piece of money might be given him. Give two or three answers for each problem.

11. A twenty-dollar bill changed to get five-dollar bills, one-dollar bills, and half-dollars?

12. A ten-dollar bill, changed to get two-dollar bills, one-dollar bills, half-dollars, and quarters?

13. A five-dollar bill, changed to get dollars, half-dollars, quarters, and dimes?

14. A two-dollar bill, changed to get half-dollars and dimes?

15. A one-dollar bill, changed to get quarters, dimes, nickels, and pennies?

16. A half-dollar, to get dimes, nickels, and pennies?

24 sheets = one quire. 20 quires = one ream.

1. How many sheets in $\frac{1}{2}$ quire? In $\frac{3}{4}$ quire?

2. How many sheets in $\frac{1}{3}$ ream? In 2 reams?

3. How many quires in 72 sheets? In 96 sheets?

4. What is the cost of a ream of paper, at 3 sheets for 1¢? At 5 sheets for 1¢?

5. A man bought one box of paper containing 2 quires and another containing 5 quires. How many sheets of paper did he buy?

6. If 12 sheets of paper cost 5¢, what is the cost of a ream at the same rate?

7. I paid $1.20 for a ream of paper. What was the cost of $\frac{1}{2}$ quire?

8. How many boxes will hold a ream of paper if each box contains $1\frac{1}{3}$ quires? $1\frac{1}{4}$ quires?

9. A man bought paper at $1.50 a ream and sold it at 12 sheets for 5¢. How much did he gain?

10. A man bought paper at $2.75 a ream and sold it at 20¢ a quire. What was his gain? If he had paid $3 a ream and sold it at 25¢ a quire, how much more would he have gained?

11. A ream of paper is divided into 40 equal portions. What part of a quire is each portion?

12. From 3 reams of paper $1\frac{1}{4}$ reams were sold at one time and 3 quires at another time. How many sheets were left? How many quires? What part of a ream?

13. After collecting damaged lots of paper, a man had 18 quires and 14 sheets of one kind; 11 quires and 5 sheets of another kind, and 10 quires and 5 sheets of a third kind. He sold it at $1.25 a ream. How much did he get for all?

1. A man leaves his home each work day at 8:11 A.M. and reaches his office at 8:27. He returns in the evening, leaving his office at 5:45 and reaching his home at 6:01. How many hours does he spend on the way in a week? In the month of January, if it has 4 Sundays?

2. A teacher leaves her home every school-day at 8:20 A.M., and reaches the schoolhouse at 8:44. She returns, leaving the schoolhouse at 3:43 P.M., and reaching her home at 4:08. How many hours does she spend on the way in 4 weeks? How many hours in the school year of 38 weeks?

3. How far does she travel in 4 weeks, if the schoolhouse is $\frac{7}{8}$ of a mile from her home? How far does she travel in a school year of 40 weeks?

4. A train leaves New York at 7:55 A.M., and reaches Washington at 1:33 P.M. It stops 11 minutes at stations on the way. What is the actual running time from New York to Washington?

5. A train leaves Chicago at 9:00 P.M., and arrives in St. Louis at 7:28 the next morning. Another train leaves Chicago at 11:30 P.M., reaching St. Louis at 8:04 next morning. In how much less time does one train run than the other, and which is the faster train?

6. A train leaves Pittsburg at 1:10 P.M., and reaches Buffalo at 8:15 the same evening. If 23 minutes are spent in stopping at stations, what is the actual running time from Pittsburg to Buffalo?

7. A second train leaves Pittsburg at 11:00 P.M., and reaches Buffalo at 7:09 next morning. How much longer is this train on the way than the one which left Pittsburg at 1:10 P.M.?

1. A lot cost $200. The house cost 12½ times as much as the lot, and the fence ½ as much as the lot. What did the house and the fence together cost?

2. The battle of Lexington was fought April 19, 1775. How many Aprils have there been from then to the present day?

3. The buildings for the Columbian Exposition were dedicated October 21, 1892. How many years, months, and days since the buildings were dedicated?

4. Nathaniel Hawthorne was born July 4, 1809. Texas was admitted to the Union Dec. 29, 1845. How old was Hawthorne when Texas became a state?

5. Daniel Webster was born January 18, 1782, and James A. Garfield was born November 19, 1831. How old was Webster when Garfield was born?

6. California was admitted as a state Sept. 9, 1850, and Oklahoma, Nov. 16, 1907. How long had California been a state when Oklahoma was admitted?

7. Massachusetts was made a state Feb. 7, 1788. How long had Massachusetts been a state when Oklahoma was admitted as a state?

8. How long was it from the Johnstown flood of May 31, 1889, to the Galveston tornado of Sept. 8, 1900?

9. How long was it from the Chicago fire of Oct. 8, 1871, to the Baltimore fire of Feb. 7, 1904?

10. How long was it from the Baltimore fire of Feb. 7, 1904, to the San Francisco earthquake and fire of Apr. 18, 1906?

11. How long was it from the beginning of the Spanish-American war of Apr. 25, 1898, to the beginning of the Russo-Japanese war of Feb. 7, 1904?

1. A grain dealer bought 25,000 bu. of wheat at 97¢ per bushel, and after 3 months, sold it for $1.12 per bushel. He paid storage charges at the rate of ½¢ per bu. a month. What was his gain?

2. A grain elevator holds 800,000 bushels. If it is kept full for 6 months, what will storage charges amount to at ¼¢ per bu. a month?

3. Six vessels carry 800,000 bu. of grain from Chicago to Buffalo. If 1 of them carries 160,000 bu., what is the average load of one of the other five?

4. Wheat weighs 60 lb. to the bushel. What is the weight, in tons, of 160,000 bushels? In hundredweight?

5. An elevator containing 645,000 bu. of grain caught fire and the grain was damaged. The grain was worth 87¢ per bu. and was insured for $250,000. What was the loss not covered by insurance?

6. The owner of the grain, after receiving the insurance money, sold the damaged grain for feed at 13¢ per bushel. What was his actual loss?

7. Corn weighs 56 pounds to the bushel. How many car loads of 20 tons each will fill a vessel that can carry 90,000 bushels?

8. At 10¢ per cwt., what will it cost to ship 90,000 bu. of corn from Chicago to New York City?

9. A vessel owner agrees to carry 125,000 bu. of corn for $3750. How much does he charge per cwt.? Per ton?

10. A builder received $127.25 for making some repairs to a house. He pays each of 2 helpers $2.50 per day for the 12 days needed to do the work. How much money remains for him?

1. Add:

1	rd.	3	yd.	2	ft.	7	rd.	4	yd.	2	ft.
3		4		1		2		3		1	
		2		2		1		2		2	
3	rd.	5	yd.	1	ft.	79	rd.	4	yd.	1	ft.
2		4		2		13		2			
		3				7		5		2	

2. Subtract:

16	rd.	5	yd.	2	ft.	32	rd.	3	yd.	2	ft.
12		3		1		15		4		1	

3. Multiply:

8	rd.	3	yd.	2	ft.	7	rd.	4	yd.	1	ft.
				3						5	

4. Divide:

2)8 rd. 4 yd. 2 ft. 4)18 rd. 4 yd. 2 ft.

5. A room is 16 ft. long and 14 ft. 6 in. wide. How many feet of picture moulding would be needed to go around it? How much would it cost at $\$.12\frac{1}{2}$ a foot?

6. One side of a square room is 4 yd., 1 ft., and 6 in. long. How many feet of border would be needed in papering the room?

7. A rectangular field 40 rd. wide is twice as long as it is wide. How many rods of fence will inclose it? If the fence is 6 wires high, how many feet of wire in it?

8. Corn rows in this field are 4 ft. apart, the first row 2 ft. from the long side. How many rows are there? The rows are 79 rd. 3 yd. long. How far would they reach, placed end to end?

1. Add 12 sq. yd., 4 sq. ft., 120 sq. in.; 4 sq. yd., 7 sq. ft., 20 sq. in.; 6 sq. yd., 9 sq. ft., 16 sq. in., and 7 sq. ft., 4 square inches.

2. Add 4 cu. yd., 9 cu. ft., 1200 cu. in.; 3 cu. yd., 400 cu. in.; 4 cu. ft., 600 cu. in.; 5 cu. yd., 12 cubic feet.

3. Add 15 gal., 3 qt., 1 pt.; 4 gal. 2 qt.; 6 qt., 1 pt.; 9 gal., 1 pint.

4. Add 3 bu., 3 pk., 7 qt.; 4 bu., 2 pk., 6 qt.; 3 pk., 4 qt.; 3 bu., 1 pk., 6 qt.; 1 pk., 2 quarts.

5. Add 2 T., 16 cwt., 80 lb.; 1 T., 8 cwt., 15 lb.; 3 T., 10 cwt., 50 lb.; 18 cwt., 90 pounds.

6. Add 4 hr., 45 min., 15 sec.; 6 hr., 30 min.; 55 min., 45 sec.; 3 hr., 30 seconds.

7. Subtract 16 sq. yd., 7 sq. ft., 44 sq. in. from 24 sq. yd., 5 sq. ft., 120 square inches.

8. Subtract 3 cu. yd., 9 cu. ft., 680 cu. in. from 12 cu. yd., 6 cu. ft., 1240 cubic inches.

9. Subtract 1 gal., 3 qt., 1 pt. from 4 gal., 2 qt., 1 pint.

10. Subtract 2 bu., 2 pk., 5 qt. from 6 bu., 3 pk., 4 quarts.

11. Subtract 2 T., 15 cwt., 15 lb. from 4 T., 12 cwt., 20 pounds.

12. Multiply 9 cu. yd., 8 cu. ft., 640 cu. in. by 4.

13. Multiply 6 gal., 3 qt., 1 pt. by 5.

14. Multiply 5 bu., 3 pk., 7 qt. by 2.

15. Multiply 2 T., 10 cwt., 60 lb. by 3.

16. Multiply 4 hr., 40 min., 30 sec. by 5.

17. Divide 16 sq. yd., 3 sq. ft., 141 sq. in. by 3.

18. Divide 19 cu. yd., 1 cu. ft., 216 cu. in. by 9.

19. Divide 26 gal., 3 qt., 2 gi. by 6.

20. Divide 12 bu., 1 pk., 3 qt. by 5.

21. Divide 12 T., 3 cwt., 50 lb. by 10.

1. A train traveled 32 mi., 120 rd., 7 yd., one hour, and 30 mi., 160 rd., 4 yd., the next. How far did it travel in the two hours?

2. One field contains 6 A., 80 sq. rd., and 40 sq. yd.; the field beside it contains 14 A., 120 sq. rd., $20\frac{1}{2}$ sq. yd. If the fields are joined into one, how much land will it contain?

3. A building covering 60 sq. yd., 6 sq. ft., 72 sq. in., is enlarged by an addition which covers 9 sq. yd., 8 sq. ft., 72 sq. in. What is the area covered by the entire building?

4. One room contains 185 cu. yd., 7 cu. ft., 192 cu. in. of air; another 172 cu. yd., 4 cu. ft., 432 cu. in.; a third 864 cu. yd., 2 cu. ft., 192 cu. in. If the air in all the rooms is entirely changed once every hour, how much air will be required in 1 hour?

5. A grocer had 2 gal., 2 qt., 1 pt. of vinegar in his store. He purchased 7 gal., 3 qt., 1 pt. more. How much had he then?

6. A farmer raised 56 bu., 3 pk. of Irish potatoes, and 24 bu., 2 pk. of sweet potatoes. How many had he in all?

7. One family used 1 T., 13 cwt., 50 lb. of ice in one summer; another family 1 T., 9 cwt., 75 lb. How much did both families use?

8. A steamship made the first half of a trip in 6 da., 10 hr., 45 min., and the return trip in 7 da., 3 hr., 15 min. 2 da., 6 hr., 30 min. were spent in port before returning. How long was the ship away from the starting-point?

9. A boy rode 10 mi., 80 rd., 4 yd. in 1 hour, and 2 mi. 120 rd., 3 yd. less the next hour. How far did he ride in the second hour?

1. A man owning a farm of 740 A. and 78 sq. rd. sold 290 A. and 98 sq. rd. How much land had he left?

2. In a bin holding 7 cu. yd. 9 cu. ft. 576 cu. in. a partition was placed separating a part holding 3 cu. yd. 18 cu. ft. 1152 cu. in. What was the capacity of the remaining part?

3. From a cistern holding 10 bbl. 14 gal. 1 qt. of water, a quantity was taken out leaving 2 bbl. 8 gal. 3 qt. in the cistern. How much was taken out?

4. From a load of grain weighing 2 T. 14 cwt. 30 lb., 1 T. 18 cwt. 80 lb. were removed. What was the weight of the remaining part?

5. A train made a trip in 14 hr. 15 min. 25 sec. Another train made the same trip in 1 hr. 50 min. 45 sec. less time. In what time was the trip made by the second train?

6. Two boats left the same port at the same time. One sailed 21 mi. 175 rd. 3 yd. while the second sailed twice as far. How far did the second one sail?

7. A building whose area was 84 sq. yd. 3 sq. ft. 72 sq. in. was torn down and another built whose area was three times that of the old one. What was the area of the new building?

8. In digging a cellar, 24 cu. yd. 3 cu. ft. 576 cu. in. of earth were excavated. It was then determined to make the cellar three times as large. How much more earth was removed? How much in all was removed?

9. A book dealer shipped 3 boxes of books. One weighed 1 cwt., 90 lb., 8 oz.; the second twice as much as the first, and the third, as much as both the others together. What was the entire weight?

(COPY OF BILL)

Chicago June 1, 1908

Albert Miller,

Bought of JOHN SAMPLE, BOOKSELLER

3	Books	@ 35	$1	05
2	Tablets	@ .10		20
4	Pencils	@ .05		20
6	Drawing Books	@ .10		60
		Total	2	05

(COPY OF RECEIPTED BILL)

Chicago, Dec. 23, 1907

Miss Edna Lane.

Bought of PRINCE'S TOY STORE

2	Dolls	@ 69	$1	38
3	Sets Toy Dishes	@ 87	2	61
1	Music Box		1	35
1½	Doz Xmas Cards	@ .54		81
1	Set Building Blocks			75
		Total	6	90

Received Payment,

Prince's Toy Store.

Copy, find amounts due on the following accounts, and receipt:

J. MANNING,

In account with D. L. PARMER, Dr.

19.....

Jan.	2	To 75 lb. Rice	@	$ .04		
"	2	" 330 lb. Sugar	@	.05		
"	7	" 50 lb. Java Coffee	@	.32		
"	7	" 45 lb. Tea	@	.60		
		Amount due, - - - -				

JAMES GILMAN,

In account with GEO. JOHNSON, Dr.

19.....

May	6	To 5 Days' Work	@	$2.50		
"	13	" 12 lb. Nails	@	.03		
"	14	" 7 Panes of Glass	@	.40		
June	11	" 10 gal. Paint	@	1.00		
"	18	" Job Work on House			275	00
		Amount due, - - - - - - -				

Make bills for the following:

1. Feb. 4, Mrs. J. K. Brown bought of White, Jones & Co.:

10	lb. Sugar	@	$.05
20	" Flour	@	.04
2	" Tea	@	.60

2. Jan. 11, L. B. Hall bought of Smith Bros.:

20	pr. Boys' Boots	@	$1.75
15	" Slippers	@	1.50
25	" Ladies' Shoes	@	2.75
25	" Rubbers	@	.50

3. June 4, Messrs. Black & Co. bought of Marshall Field & Co.:

3	bolts Velvet	@	$100.00
3	" Muslin	@	37.00
2	" Calico	@	10.00

4. May 5, L. French bought of Browne, Steele & Co.:

5	yd. Silk	@	$1.75
3	" Ribbon	@	.75
7	" Velvet	@	4.50
4	papers Pins	@	.07
2	" Needles	@	.08

5. Sept. 1, J. C. Hill bought of Birch & Son:

8	Histories	@	$1.25
15	Spellers	@	.25
9	Readers	@	.40
12	Grammars	@	.60
20	Arithmetics	@	.55

6. Oct. 21, O. F. Horn bought of Taylor & Co..

5	Coal Stoves	@	$20.00
10	Oil Stoves	@	8.00
25	lb. Nails	@	.04

Make out bills, supplying names, find the amounts, and receipt:

1.	9 lb. Ham	@	\$.16
	8 " Veal	@	.12
	12 " Mutton	@	.12½
	16 " Beef	@	.14
	4 " Pork	@	.08⅓
2.	13 lb. Dried Beef	@	\$.12
	25 " Codfish	@	.11
	16 " Mackerel	@	.06¼
	18 " Bacon	@	.12½
3.	30 yd. Cassimere	@	\$1.75
	70 Spools Thread	@	.03½
	64 yd. Sheeting	@	.12
	45 " Calico	@	.04
	5 Table Cloths	@	2.50
4.	112 bbl. Flour	@	\$6.20
	108 T. Hay	@	4.00
	250 bu. Wheat	@	.92
	130 " Corn	@	.32½
	75 " Barley	@	.85
5.	12 doz. Eggs	@	\$.12½
	12 lb. Rice	@	.04
	48 " Coffee	@	.33⅓
	15 " Butter	@	.22
	32 " Cheese	@	.14

6. Pupils may take market reports from newspapers and make problems like the above, using names of local merchants or of classmates.

1. Charles sleeps $\frac{1}{3}$ of the 24 hours; is in school $\frac{1}{4}$ of the 24 hours; works for his mother $\frac{1}{24}$ of the 24 hours, and uses $\frac{1}{12}$ of the 24 in eating. How many hours does he sleep? Go to school? Work for his mother? Use in eating? Use in other ways?

2. Tom read $\frac{1}{3}$ of a book on Sunday; $\frac{1}{6}$ on Tuesday, and $\frac{1}{9}$ on Wednesday. What part was read in these 3 days? What part was not read?

3. Tom's book had 324 pages. How many pages did he read on Sunday? On Tuesday? On Wednesday? How many pages were left unread?

4. Fred bought $1\frac{1}{2}$ dozen eggs on Monday, and 3 times as many on Tuesday. On Wednesday he sold $2\frac{1}{4}$ dozen. How many dozen had he left?

5. Felix had a small fruit-stand. He bought at different times: $2\frac{1}{3}$ dozen oranges; $1\frac{5}{6}$ dozen oranges; $3\frac{2}{3}$ dozen oranges, and $5\frac{1}{2}$ dozen oranges. How many oranges did he buy each time? How many in all? How many dozen in all?

6. A clerk in a small grocery store sold these quantities of sugar: 4 pounds 4 ounces, $3\frac{1}{2}$ pounds; 5 pounds 12 ounces; $7\frac{3}{4}$ pounds. How many ounces did he sell each time? How many pounds and parts of a pound did he sell in all?

7. A woman asked the meat-market clerk for a 6-pound roast. When he had weighed it he charged her \$.78, and the meat was 12 cents a pound. How much did it weigh? How much more did it weigh than the woman required?

8. A rug 5 yd. 1 ft. long is $\frac{3}{4}$ as wide as it is long. How many feet wide is it?

1. Mary bought $\frac{1}{2}$ lb. of chocolate creams, and Anna bought $\frac{1}{4}$ lb. more than Mary. What part of a pound did Anna buy? What part of a pound did both girls buy?

2. Charles bought $1\frac{1}{2}$ lb. of sweet potatoes one day; $1\frac{3}{4}$ lb. the next day, and $2\frac{1}{2}$ lb. another day. How many pounds of sweet potatoes did he buy in the 3 days?

3. Ellen used $1\frac{1}{3}$ yd. of ribbon for her hair; $1\frac{2}{3}$ yd. for a neck ribbon, and $3\frac{5}{8}$ yd. for a sash. How many yards of ribbon did she use in all?

4. Ross had a kite string $60\frac{1}{2}$ yd. long. He gave Carl $23\frac{3}{4}$ yd. of it. How many yards were left for Ross?

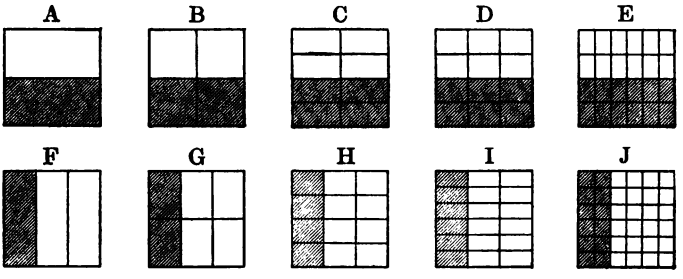
5. Grace bought $7\frac{5}{6}$ yd. of ribbon. She gave $2\frac{1}{2}$ yd. to Bessie, and $1\frac{2}{3}$ yd. to May. How many yards did Grace give away? How many yards had she left?

6. Carl had $\$5\frac{1}{2}$, and he bought a football suit for $\$4\frac{3}{4}$. What part of a dollar had he left?

7. Nellie's mother canned $3\frac{3}{4}$ pk. of pears; $1\frac{5}{8}$ pk. of peaches; $2\frac{3}{8}$ pk. of plums, and $3\frac{1}{2}$ pk. in all of different kinds of berries. How many pecks of fruit had she canned in all?

8. Four sizes of jars were on a shelf. The first size held $\frac{1}{2}$ pint; the second held $1\frac{1}{2}$ times as much as the first; the third held twice as much as the second, and the fourth held 4 times as much as the first. How much did the second hold? The third? The fourth? How much did 4 of the jars, 1 of each size, hold?

9. Walter spent $\frac{3}{4}$ of an hour studying his arithmetic; $\frac{1}{3}$ as long studying his spelling, and twice as long in studying his geography as in studying his spelling. How much time did he spend on the 3 studies?



1. Into how many parts is the square A, divided, and what is one part called? B? C? D? E?

2. Into how many parts is the square, F, divided, and what is one part called? G? H? I? J?

3. One part of A equals how many parts of B? Of C? Of D? Of E? Of G? Of I? Of J?

4. One part of F equals how many parts of G? Of H? Of I? Of J? Of D? Of E?

The number below the line of a fraction is the denominator, and indicates into how many equal parts anything is divided.

Thus, as one of the parts of B is $\frac{1}{4}$, the denominator, $\frac{}{4}$, shows that B is divided into 4 equal parts; as one part of G is $\frac{1}{6}$, the denominator, $\frac{}{6}$, shows that G is divided into 6 equal parts; etc.

The number above the line is the numerator, and indicates how many parts are taken.

Thus, in $\frac{2}{4}$ of B, the numerator, $\frac{2}{}$, shows that 2 of the 4 equal parts are taken; in $\frac{2}{6}$ of G, the numerator, $\frac{2}{}$, shows that 2 of the 6 equal parts are taken; etc.

5. What is the sum of $\frac{1}{2}$ and $\frac{1}{4}$? $\frac{1}{2}$ and $\frac{1}{8}$? $\frac{1}{2}$ and $\frac{1}{24}$? $\frac{1}{2}$ and $\frac{1}{6}$? $\frac{1}{2}$ and $\frac{1}{18}$? How did you add $\frac{1}{2}$ and $\frac{1}{8}$?

6. What must be true of the denominators of fractions in order that the fractions can be added?

1. Add $\frac{1}{2}$ and $\frac{1}{3}$. Study the square, G, p. 91.

2. In $\frac{1}{2}$ there are how many $\frac{1}{6}$'s? In $\frac{1}{3}$ there are how many $\frac{1}{6}$'s?

3. How many $\frac{1}{6}$'s are there in $\frac{1}{2}$ and $\frac{1}{3}$ together?

4. What is the difference between $\frac{1}{2}$ and $\frac{1}{4}$?

5. What is the difference between $\frac{1}{2}$ and $\frac{1}{6}$?

6. What is the difference between $\frac{1}{3}$ and $\frac{1}{6}$?

7. What is the difference between $\frac{1}{2}$ and $\frac{1}{3}$?

8. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$?

9. What is the sum of $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{8}$? $\frac{1}{3}$ and $\frac{1}{9}$?

10. What is the sum of $\frac{1}{6}$ and $\frac{1}{9}$? $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{9}$?

11. What is the difference between $\frac{1}{2}$ and $\frac{1}{9}$?

12. What is the difference between $\frac{1}{3}$ and $\frac{1}{8}$?

13. What is the difference between $\frac{1}{2}$ and $\frac{3}{8}$? Between $\frac{2}{3}$ and $\frac{1}{2}$? Between $\frac{3}{4}$ and $\frac{1}{2}$? Between $\frac{5}{8}$ and $\frac{1}{4}$? Between $\frac{5}{6}$ and $\frac{1}{3}$?

14. If $\frac{3}{4}$ of a yard of ribbon costs $21¢$, what is the cost of $\frac{1}{4}$ of a yard? The cost of $\frac{1}{2}$ of a yard equals what part of the cost of $\frac{3}{4}$ of a yard?

15. John worked $\frac{1}{2}$ of the day Monday; $\frac{3}{4}$ of the day Tuesday, and $\frac{1}{3}$ of the day Wednesday. How many days did he work altogether?

16. A girl having $\frac{3}{4}$ yard of ribbon bought $\frac{1}{8}$ yard more. What part of a yard had she then?

17. If from $\frac{7}{8}$ of a gallon of milk $\frac{1}{4}$ of a gallon is taken, what part of a gallon is left?

18. A boy studied $1\frac{1}{2}$ hours Monday, $1\frac{1}{4}$ hours Tuesday, and $1\frac{7}{8}$ hours Wednesday. How long did he study during the three days? How much longer Monday than Tuesday? How much longer Wednesday than Monday? How much longer Wednesday than Tuesday?

1. If $\frac{2}{3}$ of a yard of cloth cost \$.96, how much will $\frac{1}{6}$ of a yard cost?

2. A girl walked $\frac{2}{3}$ of a mile north, and then $\frac{1}{6}$ of a mile east. What part of a mile did she walk in all?

3. A man traveled $1\frac{1}{6}$ miles east, and returned. He then went $\frac{1}{2}$ mile west. How far did he travel?

4. From a piece of cloth $24\frac{2}{3}$ yd. long, $6\frac{1}{3}$ yd. were sold to one customer and $8\frac{1}{6}$ yd. to another. How many yards remained?

5. One box weighs $5\frac{1}{2}$ lb.; a second, 3 times as much as the first, and a third, $\frac{1}{2}$ as much as the second. What is the weight of the second box? The third box?

6. A family uses $3\frac{1}{2}$ cwt. of ice one week; $2\frac{1}{3}$ cwt. the next week. How much do they use in the 2 weeks?

7. How many jars holding $\frac{1}{3}$ of a gal. each can be filled from $3\frac{2}{3}$ gal. of water?

8. A boy attends school $\frac{5}{6}$ of the year; $\frac{1}{2}$ of his vacation is spent in the country. What part of the year does he spend in the country?

9. A milkman sells $\frac{3}{4}$ of a pt. of cream and $1\frac{1}{2}$ qt. of milk a day to each of 2 families. How much cream does he sell to both in four days? How much milk?

10. One girl was absent from school $\frac{1}{3}$ of a month; another girl was absent $\frac{1}{2}$ as long. What part of the month was the second girl absent?

11. How many bean bags, each requiring $\frac{1}{8}$ of a yd. of cloth, can be made from $\frac{1}{4}$ of a yd.? From $\frac{1}{2}$ a yd.? From $\frac{3}{4}$ of a yd.? $\frac{7}{8}$ of a yard?

12. A traveler spent $\frac{3}{4}$ yr. in England; $\frac{1}{3}$ yr. in France; $\frac{1}{6}$ yr. in Germany, and $\frac{3}{4}$ yr. in Italy. How many years did he spend in the four countries?

1. A woman used $4\frac{1}{2}$ qt. of berries; $3\frac{1}{4}$ qt. of currants, and $2\frac{1}{8}$ qt. of cherries. How many quarts of fruit did she use?

2. In making a garden, a man planted $\frac{1}{4}$ A. in cabbages; $\frac{1}{8}$ A. in peas; $\frac{1}{16}$ A. in beans, and $\frac{1}{32}$ A. in tomatoes. How much land did he plant in all?

3. A girl spent $\frac{1}{3}$ of the summer in the country; $\frac{2}{9}$ of the summer in the mountains, and the rest of it at home. What part of the summer was she at home?

4. While canning peaches, a woman cut 12 peaches into halves and 12 into thirds. How many halves were there? How many thirds? How many more thirds than halves? How many thirds equal four halves? Six halves? Eight halves?

5. A man traveled $\frac{1}{3}$ of a certain distance by boat; $\frac{4}{9}$ by train and walked $\frac{1}{2}$ the remaining distance. What part of the distance did he walk?

6. From a lot of $5\frac{1}{2}$ doz. pairs of shoes, $2\frac{1}{4}$ doz. were sold at one price, and the rest at another price. How many dozen were sold at the second price?

7. A man removed $1\frac{1}{9}$ yd. of fence from a fence $4\frac{1}{2}$ yd. long. How many yards of fence remained?

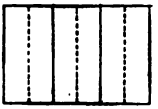
8. The area of a building was $30\frac{1}{4}$ sq. yd. An addition, $\frac{1}{3}$ as large as the building, was made. What was the area of the addition?

9. A boy spent $\frac{3}{4}$ of every year at college, for 4 years. How much time did he spend there in all?

10. A carpenter bought 528 ft. of lumber; used $\frac{1}{4}$ of it; sold $\frac{1}{3}$ of the remainder, and stored what was left. What part did he sell? What part did he keep? How many feet did he keep?

1. $\frac{1}{3} + \frac{1}{6} = ?$ $\frac{1}{3} - \frac{1}{6} = ?$

A



Show $\frac{1}{3}$ of the rectangle, A; $\frac{1}{6}$ of it.

Into how many equal parts is the rectangle divided?

What part of the rectangle is one of these parts? Two of them?

How many $\frac{1}{6}$'s are there in $\frac{1}{3}$ of the rectangle? In $\frac{1}{3} + \frac{1}{6}$ of it? In $\frac{1}{3} - \frac{1}{6}$ of it?

2. Solve, using the rectangle A, if necessary:

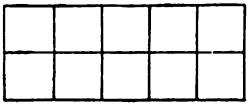
(1) $\frac{2}{3} + \frac{1}{6} = ?$ (4) $\frac{1}{3} - \frac{1}{6} = ?$ (7) $\frac{1}{2} + \frac{1}{3} = ?$

(2) $\frac{2}{3} - \frac{1}{6} = ?$ (5) $\frac{5}{6} - \frac{2}{3} = ?$ (8) $\frac{1}{2} - \frac{1}{3} = ?$

(3) $\frac{1}{3} + \frac{1}{6} = ?$ (6) $\frac{5}{6} - \frac{1}{3} = ?$ (9) $\frac{2}{3} - \frac{1}{2} = ?$

3. $\frac{2}{5} + \frac{3}{10} = ?$ $\frac{2}{5} - \frac{3}{10} = ?$

B



Show $\frac{1}{5}$ of the rectangle, B; $\frac{2}{5}$ of it; $\frac{1}{10}$ of it; $\frac{3}{10}$ of it.

Into how many equal parts is the rectangle divided?

One of these parts is what part of the rectangle?

How many $\frac{1}{10}$'s are there in $\frac{1}{5}$ of the rectangle? In $\frac{1}{5}$ of it? In $\frac{2}{5} + \frac{3}{10}$ of it? In $\frac{2}{5} - \frac{3}{10}$ of it?

4. Solve, using the rectangle B, if necessary:

(1) $\frac{3}{5} + \frac{1}{10} = ?$ (4) $\frac{4}{5} - \frac{1}{2} = ?$ (7) $\frac{1}{2} + \frac{1}{5} = ?$

(2) $\frac{2}{5} - \frac{1}{10} = ?$ (5) $\frac{1}{2} + \frac{3}{10} = ?$ (8) $\frac{1}{2} + \frac{1}{10} = ?$

(3) $\frac{2}{5} + \frac{2}{10} = ?$ (6) $\frac{7}{10} - \frac{3}{5} = ?$ (9) $\frac{1}{2} - \frac{1}{5} = ?$

5. Solve, using a divided rectangle if necessary:

(1) $\frac{1}{4} + \frac{1}{2} = ?$ (7) $\frac{5}{6} - \frac{5}{12} = ?$ (13) $\frac{4}{5} + \frac{2}{15} = ?$

(2) $\frac{3}{4} - \frac{3}{8} = ?$ (8) $\frac{3}{7} + \frac{2}{21} = ?$ (14) $\frac{3}{4} + \frac{3}{16} = ?$

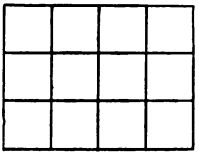
(3) $\frac{2}{3} - \frac{1}{6} = ?$ (9) $\frac{1}{6} + \frac{5}{18} = ?$ (15) $\frac{3}{4} + \frac{7}{20} = ?$

(4) $\frac{1}{5} + \frac{3}{10} = ?$ (10) $\frac{4}{9} - \frac{5}{18} = ?$ (16) $\frac{5}{7} - \frac{3}{14} = ?$

(5) $\frac{8}{9} - \frac{2}{3} = ?$ (11) $\frac{7}{8} + \frac{1}{24} = ?$ (17) $\frac{9}{10} - \frac{3}{20} = ?$

(6) $\frac{5}{8} + \frac{3}{16} = ?$ (12) $\frac{7}{9} - \frac{1}{27} = ?$ (18) $\frac{12}{13} - \frac{3}{26} = ?$

1. $\frac{3}{4} + \frac{2}{3} = ?$ $\frac{3}{4} - \frac{2}{3} = ?$



Show $\frac{1}{4}$ of the rectangle; $\frac{2}{4}$ of it; $\frac{3}{4}$; $\frac{1}{3}$; $\frac{2}{3}$.

Into how many squares is the rectangle divided? One of these squares is what part of the rectangle?

How many $\frac{1}{12}$'s are there in $\frac{1}{4}$ of the rectangle? In $\frac{3}{4}$? In $\frac{1}{3}$? In $\frac{2}{3}$? In $\frac{3}{4} + \frac{2}{3}$? In $\frac{3}{4} - \frac{2}{3}$?

2. Find the following sums and differences, using a divided rectangle when needed:

(1) $\frac{3}{5} + \frac{2}{3}$	(6) $\frac{5}{7} - \frac{2}{3}$	(11) $\frac{3}{5} + \frac{1}{4}$	(16) $\frac{5}{8} + \frac{3}{7}$
(2) $\frac{4}{5} + \frac{2}{3}$	(7) $\frac{5}{6} - \frac{2}{5}$	(12) $\frac{3}{5} + \frac{3}{7}$	(17) $\frac{5}{8} - \frac{3}{7}$
(3) $\frac{5}{6} + \frac{2}{5}$	(8) $\frac{1}{6} - \frac{1}{7}$	(13) $\frac{5}{6} - \frac{3}{5}$	(18) $\frac{3}{8} + \frac{1}{6}$
(4) $\frac{1}{6} + \frac{1}{7}$	(9) $\frac{4}{5} + \frac{2}{7}$	(14) $\frac{5}{6} + \frac{1}{7}$	(19) $\frac{3}{7} + \frac{4}{5}$
(5) $\frac{4}{5} - \frac{2}{3}$	(10) $\frac{4}{5} - \frac{2}{7}$	(15) $\frac{7}{8} - \frac{2}{3}$	(20) $\frac{6}{11} + \frac{5}{6}$

3. How may the denominator of the sum, or difference, of two fractions be found from the given denominators more quickly than by drawing and dividing a rectangle and counting its parts?

4. Compare $\frac{4}{5} + \frac{2}{3}$ with $\frac{4 \times 3}{5 \times 3} + \frac{2 \times 5}{3 \times 5}$, or $\frac{12}{15} + \frac{10}{15}$, or $\frac{22}{15}$, or $1\frac{7}{15}$.

Compare $\frac{4}{5} - \frac{2}{3}$ with $\frac{4 \times 3}{5 \times 3} - \frac{2 \times 5}{3 \times 5}$, or $\frac{12}{15} - \frac{10}{15}$, or $\frac{2}{15}$.

5. If the denominator of a fraction is multiplied by a number, what must be done to its numerator that the value of the fraction may not be changed?

6. In this way find the following sums and differences:

(1) $\frac{5}{6} + \frac{2}{5}$	(5) $\frac{3}{8} + \frac{1}{5}$	(9) $\frac{6}{10} + \frac{4}{5}$	(13) $\frac{11}{12} - \frac{8}{11}$
(2) $\frac{5}{6} - \frac{2}{5}$	(6) $\frac{3}{8} - \frac{1}{5}$	(10) $\frac{4}{5} - \frac{2}{7}$	(14) $\frac{11}{12} + \frac{9}{11}$
(3) $\frac{4}{5} + \frac{1}{3}$	(7) $\frac{7}{9} + \frac{1}{4}$	(11) $\frac{11}{12} - \frac{3}{5}$	(15) $\frac{7}{16} - \frac{2}{9}$
(4) $\frac{4}{5} - \frac{1}{3}$	(8) $\frac{7}{9} - \frac{1}{4}$	(12) $\frac{11}{12} + \frac{2}{5}$	(16) $\frac{12}{19} + \frac{15}{20}$

1. Solve:

(1) $2\frac{1}{2} + 1\frac{1}{4} = ?$ (4) $5\frac{2}{3} + 2\frac{1}{6} = ?$ (7) $8\frac{3}{4} - 6\frac{1}{2} = ?$

(2) $3\frac{1}{4} + 1\frac{1}{12} = ?$ (5) $4\frac{3}{8} + 1\frac{1}{4} = ?$ (8) $9\frac{3}{5} - 7\frac{1}{10} = ?$

(3) $8\frac{3}{4} + 2\frac{1}{8} = ?$ (6) $6\frac{1}{2} - 3\frac{1}{4} = ?$ (9) $4\frac{1}{2} - 2\frac{1}{6} = ?$

A number like $8\frac{3}{4}$, which is made up of a whole number and a fraction, is called a mixed number.

To add or subtract mixed numbers, first add or subtract the fractions, then add or subtract the whole numbers, and then add the two results.

2. (1) $9\frac{3}{4} + 6\frac{2}{3} = ?$

$9\frac{3}{4} = 9\frac{9}{12}$

$6\frac{2}{3} = 6\frac{8}{12}$

$15\frac{17}{12} = 15 + 1\frac{5}{12} = 16\frac{5}{12}$

(2) $9\frac{3}{4} - 6\frac{4}{5} = ?$

$9\frac{3}{4} = 9\frac{15}{20} = 8\frac{35}{20}$

$6\frac{4}{5} = 6\frac{16}{20} = 6\frac{16}{20}$

$2\frac{19}{20}$

Why is the minuend, $9\frac{15}{20}$, in (2) changed to $8\frac{35}{20}$?

3. Solve in like manner:

(1) $6\frac{2}{3} + 2\frac{4}{5} = ?$ (5) $8\frac{1}{3} - 5\frac{2}{3} = ?$ (9) $18\frac{7}{8} + 6\frac{9}{10} = ?$

(2) $5\frac{1}{8} + 3\frac{2}{3} = ?$ (6) $10\frac{1}{2} - 8\frac{4}{5} = ?$ (10) $20\frac{2}{5} - 15\frac{4}{5} = ?$

(3) $4\frac{1}{4} - 2\frac{1}{2} = ?$ (7) $12\frac{1}{2} - 6\frac{2}{3} = ?$ (11) $33\frac{1}{3} - 16\frac{2}{3} = ?$

(4) $12\frac{2}{3} - 6\frac{3}{4} = ?$ (8) $15\frac{5}{6} + 7\frac{1}{3} = ?$ (12) $66\frac{2}{3} - 6\frac{1}{4} = ?$

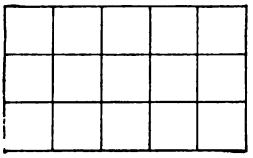
4. Henry Adams has $5\frac{3}{4}$ A. of land in one field and $10\frac{5}{8}$ A. in another. How much land has he in both fields?

5. Fred drives $5\frac{3}{4}$ miles on Monday; $3\frac{3}{8}$ miles on Tuesday, and $6\frac{1}{2}$ miles on Wednesday. How far does he drive in the three days?

6. One man works $4\frac{1}{2}$ days and another $3\frac{2}{3}$ days. How much longer does the first work than the second?

7. From a bin containing $5\frac{5}{8}$ bushels of corn $3\frac{1}{4}$ bushels were removed at one time, and $1\frac{1}{2}$ bushels at another. How many bushels then remained in the bin?

1. $2 \times \frac{1}{3}$ of the rectangle equals what part of it?



2. $3 \times \frac{1}{5} = ?$ $2 \times \frac{2}{5} = ?$ $4 \times \frac{2}{15} = ?$

Such an expression as $\frac{2}{3} \times \frac{4}{5}$ means $\frac{2}{3}$ times $\frac{4}{5}$, but it may be read "$\frac{2}{3}$ of $\frac{4}{5}$."

3. $\frac{2}{3} \times \frac{4}{5}$, or $\frac{2}{3}$ of $\frac{4}{5} = ?$

Show $\frac{1}{3}$ of the rectangle; $\frac{4}{5}$ of it; $\frac{1}{3}$ of $\frac{4}{5}$ of it.

What part of the whole rectangle is one of the small squares? How many of these squares are there in $\frac{1}{3}$ of $\frac{4}{5}$ of the rectangle? What part of the whole rectangle is $\frac{1}{3}$ of $\frac{4}{5}$ of it? $\frac{2}{3}$ of $\frac{4}{5}$ of it? $\frac{2}{3} \times \frac{4}{5} = ?$

4. Solve: (1) $\frac{1}{3} \times \frac{1}{5} = ?$ (2) $\frac{1}{2} \times \frac{5}{6} = ?$ (3) $\frac{2}{3} \times \frac{1}{6} = ?$

5. $\frac{3}{4} \times \frac{5}{6} = ?$

Divide any rectangle into 6 equal parts by lines running across it one way, and into 4 equal parts by lines running across it the other way. Show first, what part of the whole rectangle $\frac{1}{4}$ of $\frac{5}{6}$ of it is, and then what part $\frac{3}{4}$ of $\frac{5}{6}$ of it is. $\frac{3}{4}$ of $\frac{5}{6} = ?$ Show on a rectangle that $\frac{1}{2}\frac{5}{4} = \frac{5}{8}$.

6. Solve:

(1) $\frac{2}{3} \times \frac{4}{7} = ?$ (3) $\frac{6}{7} \times \frac{8}{11} = ?$ (5) $\frac{3}{4} \times \frac{7}{8} = ?$

(2) $\frac{2}{5} \times \frac{1}{3} = ?$ (4) $\frac{5}{12} \times \frac{7}{8} = ?$ (6) $\frac{3}{2} \times \frac{5}{7} = ?$

7. Compare the product of the numerators of the fractions in each exercise in problem 6 with the numerator of the result. Compare the product of the denominators of the fractions in each exercise in problem 6 with the denominator of the result.

How may the product of two fractions be found quickly?

8. Solve by the quicker method:

(1) $\frac{5}{6} \times \frac{7}{8} = ?$ (3) $\frac{6}{7} \times \frac{8}{13} = ?$ (5) $\frac{4}{5} \times \frac{4}{5} = ?$

(2) $\frac{5}{7} \times \frac{11}{12} = ?$ (4) $\frac{12}{13} \times \frac{11}{13} = ?$ (6) $\frac{7}{8} \times \frac{17}{19} = ?$

1. How many $\frac{3}{16}$-lb. packages can be made from $\frac{15}{16}$ of a pound of pepper?

2. John worked $\frac{3}{5}$ of a week and Joe $\frac{4}{5}$ of a week at the same weekly salary. What part of Joe's pay for the week did John's pay equal?

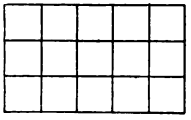
3. Compare $\frac{3}{5} \div \frac{4}{5}$, with $\frac{\frac{3}{5}}{\frac{4}{5}}$, or $\frac{3 \text{ fifths}}{4 \text{ fifths}}$, or $\frac{3}{4}$.

4. Solve:

(1) $\frac{4}{5} \div \frac{2}{5} = ?$ (3) $\frac{7}{9} \div \frac{2}{9} = ?$ (5) $\frac{1}{2} \div \frac{1}{4} = ?$

(2) $\frac{8}{9} \div \frac{2}{9} = ?$ (4) $\frac{7}{11} \div \frac{2}{11} = ?$ (6) $\frac{3}{4} \div \frac{1}{8} = ?$

5. $\frac{3}{5} \div \frac{2}{3} = ?$



Divide a rectangle into 5 equal parts by lines running across it one way, and into 3 equal parts by lines running across it the other way.

Into how many equal parts is the rectangle then divided?

What part of the whole rectangle is one of them?

To how many $\frac{1}{15}$'s is $\frac{3}{5}$ equal? $\frac{2}{3}$? $\frac{9}{15} \div \frac{10}{15} = ?$ $\frac{3}{5} \div \frac{2}{3} = ?$

6. How may two fractions be changed to equivalent fractions having the same denominator, in a quicker way than by using a divided rectangle? (Prob. 3, p. 98.)

SOLUTION:

7. $\frac{2}{3} \div \frac{3}{4} = ?$ $\frac{2}{3} = \frac{8}{12}$. $\frac{3}{4} = \frac{9}{12}$. $\frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$.

8. Solve:

(1) $\frac{1}{6} \div \frac{2}{3} = ?$ (7) $\frac{12}{13} \div \frac{1}{2} = ?$ (13) $\frac{5}{6} \div \frac{7}{9} = ?$

(2) $\frac{7}{18} \div \frac{2}{9} = ?$ (8) $\frac{3}{7} \div \frac{4}{5} = ?$ (14) $\frac{1}{4} \div \frac{5}{6} = ?$

(3) $\frac{5}{7} \div \frac{2}{21} = ?$ (9) $\frac{1}{2} \div \frac{6}{11} = ?$ (15) $\frac{3}{5} \div \frac{1}{3} = ?$

(4) $\frac{7}{8} \div \frac{3}{4} = ?$ (10) $\frac{5}{7} \div \frac{2}{3} = ?$ (16) $\frac{9}{10} \div \frac{2}{15} = ?$

(5) $\frac{8}{9} \div \frac{2}{5} = ?$ (11) $\frac{3}{4} \div \frac{4}{5} = ?$ (17) $\frac{7}{9} \div \frac{4}{27} = ?$

(6) $\frac{6}{7} \div \frac{3}{5} = ?$ (12) $\frac{11}{12} \div \frac{7}{8} = ?$ (18) $\frac{21}{25} \div \frac{3}{4} = ?$

In a certain town hard coal costs $7.25 a ton during April. The price then rises 10 cents a ton on the first day of each following month for 5 months, and remains at this figure until the first day of the next April.

1. What does coal cost per ton in this town during May? During June? During July? August? September? October? November? December? January? February? March?

2. If a man buys hard coal in April what must he pay for 2 tons? For 3 tons? For 5 tons? For 6 tons? For 8 tons?

3. How much must a man pay for 3 tons of coal, if he buys it some time in May? In June? In July? August? September? October? December? January? February?

4. How many tons of hard coal could a man buy during April for $14.50? For $29? For $58? For $72.25? For $87?

5. How many tons of hard coal could a man buy during May for $14.70? For $29.40? For $44.10? For $88.20? For $58.80? For $132.80?

6. The retail dealer pays $1 less per ton than he sells the coal for to the people who use it. How much must he pay for 1 car load of 40 tons in April? In May? In June? In December?

7. The retail dealer bears whatever loss occurs while the car is being brought from the mines. How much does he make on the carload of 40 tons, weighed at the mines, if the car actually contained only 38 tons when it reached him?

8. How much money does a man save who invests in 10 tons of coal in April to be burned in July? In October? In November?

1. What part of a dollar is 50¢? 25¢? 10¢? 1¢?

2. How do you write 1¢ as a fraction of a dollar? How do you write it with the dollar sign ($)?

3. How do you write 10¢ as a fraction of a dollar? With the dollar sign?

4. How do you write 25¢ as a fraction of a dollar? With the dollar sign? 50¢ as a fraction of a dollar? With the dollar sign?

5. Write as hundredths (100ths) of a dollar:

$.01 $.07 $.15 $.25 $.39 $.75 $.87

6. Write the following as 100ths, and as tenths (10ths) of a dollar:

$.10 $.30 $.50 $.70 $1.10 $1.50 $2.60

The period used between the figures in writing dollars and cents is called the decimal point.

7. Write each of the following as a common fractional part of a dollar, and also as a decimal part of a dollar:

1¢; 10¢; 25¢; 50¢; 75¢; 120¢; 150¢.

Numbers when written with the decimal point are read exactly as when written as fractions.

$\frac{3}{100}$ of a dollar is read *three-hundredths* of a dollar.

.03 of a dollar is read *three-hundredths* of a dollar.

$\frac{25}{100}$ of a dollar is read *twenty-five-hundredths* of a dollar.

.25 of a dollar is read *twenty-five-hundredths* of a dollar.

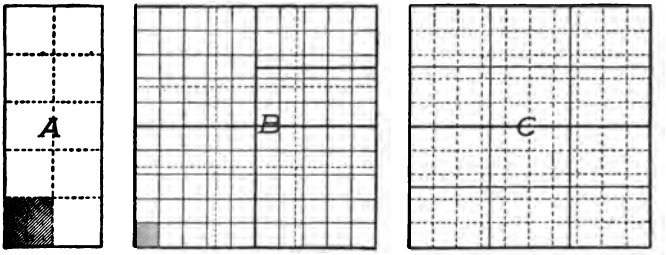
$\frac{5}{10}$ of a dollar is read *five-tenths* of a dollar.

.5 of a dollar is read *five-tenths* of a dollar.

8. Read the following numbers as parts of a dollar:

.05 .18 .45 .06 .23 .75 .3 .02 .6 .72 .93 .15 .8 .9 .44 .12

9. Write each of the decimal fractions in problem 8 in the form of a common fraction.



1. What part of the rectangle, A, is shaded? Not shaded?

2. Show from A how many tenths (10ths) there are in $\frac{1}{2}$. In $\frac{2}{2}$. In $\frac{1}{5}$. In $\frac{2}{5}$. In $\frac{3}{5}$. In $\frac{4}{5}$. In $\frac{5}{5}$.

3. Write these fractions as 10ths: $\frac{1}{2}$ $\frac{2}{2}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$

4. What part of the square, B, is shaded? Not shaded?

5. Show from B how many hundredths (100ths) there are in 1. In $\frac{1}{2}$. In $\frac{2}{2}$. In $\frac{1}{4}$. In $\frac{3}{4}$.

6. Show how many 100ths there are in $\frac{1}{5}$. $\frac{2}{5}$. $\frac{3}{5}$. $\frac{4}{5}$. $\frac{5}{5}$.

7. Write as 100ths: $\frac{1}{2}$ $\frac{2}{2}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{7}{10}$

Another way of writing $\frac{2}{10}$, $\frac{3}{10}$, $\frac{5}{10}$, is .2, .3, .5.

Another way of writing $\frac{25}{100}$, $\frac{45}{100}$, $\frac{75}{100}$, and so forth, is .25, .45, .75, and so forth.

When 10ths, or 100ths are written in the second way they are called *decimal fractions*, or *decimals*.

The fractions are said to be written decimally.

The decimal, .7, is read "7 *tenths*," and .37 is read "37 *hundredths*."

8. Write the following fractions decimally, and then read them:

$\frac{3}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{12}{10}$ $\frac{20}{100}$ $\frac{35}{100}$ $\frac{50}{100}$ $\frac{66}{100}$ $\frac{78}{100}$ $\frac{97}{100}$

9. Write the following decimals as common fractions, and then read them:

.4 .9 .13 .18 .26 .48 .67 .50 .25 .79 .86

1. $\frac{1}{3}$ of the square, B, (page 102) equals how many 100ths of B? $33\frac{1}{3}$ hundredths is written $.33\frac{1}{3}$.

2. $\frac{2}{3}$ of the square, B, equals how many 100ths of B?

3. Write $66\frac{2}{3}$ hundredths without the word, *hundredths*.

4. How many 100ths of B equals $\frac{1}{8}$ of B? $\frac{2}{8}$ of B? $\frac{3}{8}$ of B? $\frac{5}{8}$ of B? $\frac{7}{8}$ of B?

5. Write the following common fractions as decimals:

$$\frac{33\frac{1}{3}}{100} \quad \frac{66\frac{2}{3}}{100} \quad \frac{12\frac{1}{2}}{100} \quad \frac{37\frac{1}{2}}{100} \quad \frac{62\frac{1}{2}}{100} \quad \frac{87\frac{1}{2}}{100}$$

6. $\frac{1}{6}$ of the square, C (page 102), equals how many 100ths of C?

7. $\frac{5}{6}$ of the square C, equals how many 100ths of C?

8. Write $16\frac{2}{3}$, and $83\frac{1}{3}$ decimally.

9. Show from C how many 100ths $\frac{1}{12}$ equals; $\frac{5}{12}$; $\frac{7}{12}$, $\frac{11}{12}$.

10. Write the following decimals as common fractions:

$.27 \quad .33 \quad .67\frac{1}{2} \quad .72\frac{1}{3} \quad .22\frac{1}{2} \quad .44\frac{4}{9} \quad .77\frac{7}{9}$

11. Write as decimals, and then read:

$$\frac{8\frac{1}{3}}{100} \quad \frac{14\frac{2}{7}}{100} \quad \frac{41\frac{2}{3}}{100} \quad \frac{42\frac{6}{7}}{100} \quad \frac{58\frac{1}{3}}{100} \quad \frac{71\frac{3}{7}}{100} \quad \frac{91\frac{2}{3}}{100}$$

12. Show how many 100ths $\frac{1}{9}$ of the square, B, equals.

13. How many 100ths of B do these equal:

$\frac{1}{9} \quad \frac{2}{9} \quad \frac{3}{9} \quad \frac{1}{3} \quad \frac{4}{9} \quad \frac{5}{9} \quad \frac{2}{3} \quad \frac{6}{9} \quad \frac{7}{9} \quad \frac{8}{9} \quad \frac{9}{9}$

14. Write the decimal equivalents of the following:

$$\frac{11\frac{1}{9}}{100} \quad \frac{22\frac{2}{9}}{100} \quad \frac{33\frac{1}{3}}{100} \quad \frac{44\frac{4}{9}}{100} \quad \frac{55\frac{5}{9}}{100} \quad \frac{66\frac{2}{3}}{100} \quad \frac{77\frac{7}{9}}{100}$$

15. Write the following decimals as common fractions:

$.08\frac{1}{3} \quad .15\frac{1}{2} \quad .14\frac{1}{3} \quad .38\frac{1}{3} \quad .77\frac{1}{7} \quad .85\frac{3}{4} \quad .20\frac{5}{8}$

16. Give the decimal equivalents of

$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{3}{4} \quad \frac{2}{3} \quad \frac{1}{5} \quad \frac{2}{5} \quad \frac{3}{5} \quad \frac{4}{5} \quad \frac{1}{12} \quad \frac{5}{12} \quad \frac{7}{12}$

1. Express the following common fractions in the form of decimal fractions:

$\frac{3}{100}$ bu. $\frac{7}{10}$ ft. $\frac{6}{10}$ hr.

$\frac{4}{10}$ gal. $\frac{25}{100}$ ton. $\$\frac{37}{100}$

2. Write as decimal fractions the following:

$\frac{94}{100}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{9}{10}$	$\frac{6}{10}$
$\frac{7}{10}$	$\frac{86}{100}$	$\frac{15}{100}$	$\frac{12}{100}$	$\frac{2}{100}$
$\frac{18}{100}$	$\frac{19}{100}$	$\frac{9}{100}$	$\frac{6}{100}$	$\frac{2}{10}$

3. .25 of a dollar equals how many cents?

4. A farmer sold .3 of a flock of 350 sheep. How many did he sell?

5. A piece of cloth 10 yd. long was sold to two customers, one buying .5 of it, the other buying the rest. How many yards did each one take?

6. A boy traveled 100 miles, going .2 of the way by boat. How many miles did he go by boat?

7. If .07 of the contents of a cistern holding 500 gal. were drawn off, how many gallons were drawn off?

8. A man bought .75 of a quantity of grain measuring 400 bu. How many bu. did he buy?

$4\frac{7}{10}$, or 4.7 is read four *and* seven tenths.

$204\frac{9}{100}$, or 204.09 is read two hundred four *and* nine hundredths.

$234\frac{69}{100}$, or 234.69 is read two hundred thirty-four *and* sixty-nine hundredths.

9. What name is given to any figure written in the first place to the right of the decimal point? In the second place?

10. Read the following:

8.3	27.32	30.03	7.07	201.05
14.17	643.24	500.5	93.94	330.33

1. A man bought a ton of hard coal for \$6.35, a ton of soft coal for \$4.50, and a load of kindling for \$1.50. How much did the coal and kindling cost?

2. James lives .75 mi. north, and Fred, .35 mi. south of the schoolhouse. How far apart do the boys live?

3. Find the cost of 5 barrels of flour at \$4.75 a barrel.

4. One street car line has 4.85 miles of track; another 6.55 miles, and a third 7.5 miles. How many miles of track are there?

5. A farm is divided as follows: cornfield, 66.75 acres; oatfield, 42.6 acres; meadow, 24.5 acres; grove, 20.25 acres; vegetable garden, 4.4 acres; building lot, 1.5 acres. How many acres are there in the farm?

6. A bushel of potatoes contains 47.34 lb. water, 10.8 lb. starch, .54 lb. ash, 1.26 lb. proteid, and .06 lb. fat. How much does a bushel of potatoes weigh?

47.34 lb.	Begin on the right and add as with
10.80 "	whole numbers.
.54 "	Place the decimal point (.) in the sum
1.26 "	just beneath the points in the
.06 "	numbers to be added (addends).

7. Add:

(1)	(2)	(3)	(4)
623.21	4561.38	685.329	365.250
68.75	28.17	27.602	.168
90.66	191.76	7.001	90.027
8.32	9.68	.868	106.995
4.00	3125.88	12.789	78.200
67.75	.98	1.896	9.375

TABLE SHOWING FOOD PARTS

Food	Water	Fat	Ash	Proteid	Starch
Oatmeal	.72 lb.	.73 lb.	.19 lb.	1.56 lb.	6.80 lb.
Corn-meal	1.29 "	.22 "	.07 "	.89 "	7.53 "
Rye-meal	1.27 "	.09 "	.08 "	.71 "	7.85 "
Rice	1.23 "	.03 "	.04 "	.78 "	7.92 "
Macaroni	1.08 "	.16 "	.30 "	1.17 "	7.29 "

1. How much does the water in the 5 foods weigh? The fat? The ash? The proteid? The starch?

2. How much does the oatmeal weigh? The corn-meal? The rye-meal? The rice? The macaroni?

3. What is the weight of all the parts of problem 1?

4. Ten pounds of apples contain 8.46 pounds of water and 10 pounds of bananas contain 7.53 pounds of water. How much more water is there in 10 pounds of apples than in 10 pounds of bananas?

8.46 lb.
7.53 "

First, subtract as with whole numbers. Place the decimal point in the difference just beneath the points in the minuend and subtrahend.

5. Subtract:

(1)	(2)	(3)	(4)
86.3	60.9	35.16	30.16
7.9	14.7	21.87	3.95
(5)	(6)	(7)	(8)
201.06	108.08	476.00	806.30
99.98	69.89	96.99	89.92
(9)	(10)	(11)	(12)
36.004	1008.902	101.010	600.006
9.735	86.967	90.909	589.099

1. A pound of butter contains 15.6 oz. of fat. How many ounces of fat are there in 3 pounds of butter?

2. In one pound of eggs there are 1.79 oz. of shell. How many ounces of shell are there in 5 pounds of eggs?

3. How many minutes in .05 hour?

4. A lot is 125 feet deep and 12.5 feet wide. What is its area?

5. A square-cornered box is .7 of a foot long, .5 of a foot wide and .3 of a foot deep. What is the area of the top? Of the side? Of the end? Of the entire surface?

6. Multiply:

(1) $326 \times 10 = ?$ (4) $326 \times 100 = ?$ (7) $326 \times 1000 = ?$
(2) $32.6 \times 10 = ?$ (5) $32.6 \times 100 = ?$ (8) $32.6 \times 1000 = ?$
(3) $3.26 \times 10 = ?$ (6) $3.26 \times 100 = ?$ (9) $3.26 \times 1000 = ?$

7. How may a number be multiplied quickly by 10? By 100? By 1000?

8. Multiply:

(1)	(2)	(3)	(4)	(5)	(6)
68	68	68	6.8	6.8	6.8
27	2.7	.27	27	2.7	.27
476	476	476	476	476	476
136	136	136	136	136	136
1836	183.6	18.36	183.6	18.36	1.836

9. In problem 8, how many decimal figures are there in the product of (2)? Of (3)? Of (4)? Of (5)? Of (6)?

10. In problem 8, how many decimal figures are there in both the multiplicand and multiplier of (2)? Of (3)? Of (4)? Of (5)? Of (6)?

11. How may one quickly tell how many decimal places there will be in the product of two decimals?

1. Find the product of 1235×321, and write the answers to the following without multiplying:

(1) $1235 \times 32.1 = ?$
(2) $1235 \times 3.21 = ?$
(3) $123.5 \times 32.1 = ?$
(4) $12.35 \times 3.21 = ?$
(5) $1.235 \times 321 = ?$
(6) $12.35 \times .321 = ?$

2. Solve:

(1) $32 \times 2.5 = ?$
(2) $45 \times 3.8 = ?$
(3) $63 \times .28 = ?$
(4) $3.9 \times 4.5 = ?$
(5) $8.5 \times .48 = ?$
(6) $28.6 \times 32.5 = ?$
(7) $2.86 \times 3.56 = ?$
(8) $28.6 \times 3.56 = ?$

3. Solve and express the quotients in decimal form:

(1) $253 \div 10 = ?$

$$10\overline{)253}$$
$$25.3$$

(2) $29 \div 10 = ?$
(3) $907 \div 10 = ?$
(4) $83 \div 10 = ?$

(5) $253 \div 100 = ?$

$$100\overline{)253}$$
$$2.53$$

(6) $843 \div 100 = ?$
(7) $2601 \div 100 = ?$
(8) $277 \div 100 = ?$

4. Answer the following:

(1) $\frac{1}{4}$ of 16.64 lb. $= ?$
(2) $\frac{1}{5}$ of \$2.25 $= ?$
(3) $\frac{1}{8}$ of 9.76 hr. $= ?$
(4) $94.5 \div 3 = ?$
(5) $9.45 \div 9 = ?$
(6) $.945 \div 27 = ?$

5. Divide:

(1) 864 by 36
(2) 86.4 by 36
(3) 86.4 by 3.6
(4) 8.64 by 3.6
(5) 2331 by 63
(6) 233.1 by 63
(7) 233.1 by 6.3
(8) 23.31 by 6.3

6. Solve:

(1) $86.4 \div 2.4 = ?$
(2) $140.4 \div 5.4 = ?$
(3) $425.6 \div 7.6 = ?$
(4) $101.52 \div 3.6 = ?$
(5) $25.16 \div .74 = ?$
(6) $2.491 \div .53 = ?$
(7) $13.312 \div 4.16 = ?$
(8) $83.781 \div 2.61 = ?$

1. What part of a dollar is 1 cent? 9 cents? 31 cents?

Another term for hundredths is percent.

1 hundredth of a dollar = 1 percent of a dollar.

9 hundredths of a dollar = 9 percent of a dollar.

Instead of the word, percent, the sign, %, is often used.

9 percent of a dollar may be written 9 % of a dollar.

31 percent of a dollar may be written 31 % of a dollar.

Hundredths of anything may be expressed in any of the following ways:

$\frac{17}{100}$ bu., .17 bu., 17 percent of a bu. or 17 % of a bu.

$\frac{3}{100}$ lb., .03 lb., 3 percent of a lb. or 3 % of a lb.

2. Write first with the percent sign, then as decimals:

$\frac{8}{100}$	$\frac{6}{100}$	$\frac{13}{100}$	$\frac{33}{100}$
$\frac{40}{100}$	$\frac{18}{100}$	$\frac{14}{100}$	$\frac{12}{100}$
$\frac{25}{100}$	$\frac{20}{100}$	$\frac{50}{100}$	$\frac{16}{100}$
$\frac{75}{100}$	$\frac{60}{100}$	$\frac{45}{100}$	$\frac{66}{100}$

3. Write first as common, then as decimal fractions:

10%	25%	64%	$12\frac{1}{2}$%
15%	20%	53%	$33\frac{1}{3}$%
3%	50%	2%	$16\frac{2}{3}$%
5%	75%	45%	$66\frac{2}{3}$%

4. Write first as hundredths, then with the sign, %:

.8	.25	.7	.09
.5	.14	.04	.85
.16	.75	.65	.01
.33	3.1	2.25	1.03

5. Write first as decimals, then with the sign, %:

$\frac{1}{10}$	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{1}{4}$
$\frac{1}{5}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{1}{8}$
$\frac{1}{2}$	$\frac{2}{5}$	$\frac{9}{10}$	$\frac{3}{8}$
$\frac{1}{20}$	$\frac{3}{20}$	$\frac{4}{5}$	$\frac{5}{8}$

Answer all you can orally

1. How many cents in $\frac{1}{4}$ of a dollar? In $\frac{1}{2}$ dollar? $\frac{3}{4}$ dollar? $\frac{1}{8}$ dollar? $\frac{5}{8}$ dollar? $\frac{7}{8}$ dollar?

2. What percent of a dollar is $\frac{1}{4}$ of a dollar? $\frac{3}{4}$? $\frac{1}{2}$? $\frac{1}{10}$? $\frac{1}{20}$? $\frac{3}{10}$? $\frac{3}{20}$? $\frac{1}{8}$? $\frac{5}{8}$? $\frac{3}{8}$? $\frac{7}{8}$?

3. Instead of one dollar, find the same parts and percents as in problem 2 of one gallon; of one bushel; of any unit; of \$8; of \$24; of 40 marbles; of 84 acres.

4. What fractional part is 25% of any number? 10% of it? 5%? 15%? 50%? 75%? $12\frac{1}{2}$%? 20%? $37\frac{1}{2}$%? 40%?

5. How many cents is $\frac{1}{6}$ of a dollar? $\frac{5}{6}$ of a dollar? $\frac{1}{3}$ of a dollar? $\frac{2}{3}$ of a dollar?

6. What percent of a dollar is $\frac{1}{6}$ dollar? $\frac{5}{6}$ dollar?

7. What percent of a foot is $\frac{1}{6}$ of a foot? $\frac{5}{6}$ of a foot? $\frac{1}{3}$ of a foot? $\frac{2}{3}$ of a foot?

8. What percent of a yard is $\frac{1}{6}$ of a yard? $\frac{5}{6}$ of a yard? $\frac{1}{3}$ of a yard? $\frac{2}{3}$ of a yard?

9. Find $16\frac{2}{3}$% of 18 pigeons; of 24 marbles; of 30 boys. Find $83\frac{1}{3}$% of the same; $33\frac{1}{3}$%; $66\frac{2}{3}$%.

Parts as percents.

10. What is $\frac{1}{2}$ of 8? What is 50% of 8?

11. What is $\frac{1}{4}$ of 12? What is 25% of 12?

12. What is $\frac{2}{4}$, or $\frac{1}{2}$ of 12? What is 50% of 12?

13. What is $\frac{3}{4}$ of 12? What is 75% of 12?

14. What is $\frac{1}{3}$ of 12? What is $33\frac{1}{3}$% of 12?

15. What is $\frac{2}{3}$ of 18? What is $66\frac{2}{3}$% of 18?

16. What is $\frac{1}{5}$ of 40? What is 20% of 40?

17. What is $\frac{2}{5}$ of 40? What is 40% of 40?

18. What is $\frac{1}{10}$ of 60? What is 10% of 60?

19. What is $\frac{3}{10}$ of 60? What is 30% of 60?

1. What is $\frac{1}{8}$ of 24? What is $12\frac{1}{2}\%$ of 24?

2. What is $\frac{3}{8}$ of 16? What is $37\frac{1}{2}\%$ of 16?

3. What is $\frac{1}{20}$ of 80? What is 5% of 80?

4. $\frac{1}{3}$ is what percent? $\frac{1}{4}$ is what %? $\frac{1}{5}$ is what %?

5. $\frac{1}{10}$ is what percent? $\frac{1}{2}$ is what %? $\frac{1}{8}$ is what %?

6. $\frac{2}{3}$ is what percent? $\frac{3}{10}$ is what %?

7. Give a quick way of finding 50% of a number; 25% of a number; 20% of a number; $33\frac{1}{3}\%$ of a number; 10% of a number; $12\frac{1}{2}\%$ of a number.

8. Give a quick way of finding 75% of a number; $66\frac{2}{3}\%$ of a number; 40% of a number; $37\frac{1}{2}\%$ of a number.

Finding the percentage.

9. What number is 50% of 4? 50% of 2? 50% of 10? 50% of 16? 50% of 12? 50% of 28?

10. What number is 25% of 4? 25% of 12? 25% of 16? 25% of 28? 25% of 32? 25% of 40?

11. Give 75% of 4; of 8; of 20; of 36; of 40; of 6; of 14.

12. Give $33\frac{1}{3}\%$ of 3; of 12; of 15; of 24; of 39; of 48; of 42; of 4.

13. Give $66\frac{2}{3}\%$ of 3; of 12; of 15; of 24; of 48; of 4.

14. Give 10% of 20; of 30; of 80; of 25; of 35; of 42; of 86.

15. Give 30% of 20; of 30; of 80; of 10; of 25; of 42;

16. Give 40% of 5; 10; 15; 25; 35; 45; 75.

17. 50% of Fred's money is 15 cents. How much money has he?

18. 75% of the milk one family uses in a week is 9 quarts. How much milk is used? What part of the quantity used is 9 quarts?

19. $33\frac{1}{3}\%$ of the horses in a livery stable is 21. How many horses in all? What part of the whole number of horses is 21 horses?

Decimals as percents.

1. What is .10 of 30? What is 10% of 30?

2. What is .20 of 30? What is 20% of 30?

3. What is .30 of 40? What is 30% of 40?

4. What is .50 of 20? What is 50% of 20?

5. What is .25 of 8? What is 25% of 8?

6. What is .75 of 8? What is 75% of 8?

7. What is .05 of 100? What is 5% of 100?

8. What is .04 of 100? What is 4% of 100?

9. .50 is what percent? .40 is what percent?

10. .15 is what percent? .75 is what percent?

11. .25 is what percent? .33⅓ is what percent?

12. .37½ is what percent? .87½ is what percent?

13. In an orchard of 150 trees, 20% are pear trees. How many pear trees are there?

14. A dairyman had 90 lb. of butter and sold 30% of it. How many pounds of butter did he sell?

15. A man paid $45 for a bicycle and sold it at a gain of 25%. How much did he gain?

16. 300 lb. of potatoes contain 60 lb. of starch. What percent of potato is starch?

17. A farm of 160 acres contains a meadow of 32 acres. What percent of the farm is meadow?

18. In 50 lb. of oatmeal there are 30 lb. of starch. What percent of oatmeal is starch?

19. About 80% of rice is starch. About how much starch is there in 25 lb. of rice?

20. Willis bought a collection of canceled stamps at 25% of their face value. The face value of the collection was 264 cents. How much did Willis pay for the collection?

PART II

§1. Scale Drawing. *Answer all you can orally*

1. The scale of the drawing of the picture frame is 1″ : 4″, or $\frac{1}{4}$. How wide is the frame? How long?

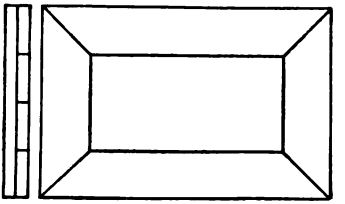


FIGURE 1

2. How long and how wide is the place for the picture? Answer question 1 for a scale of $\frac{1}{12}$; of $\frac{1}{10}$.

3. What is the scale of a drawing in which a line 1 in. long represents 1 ft.? In which 1 in. represents 40 ft.? 100 feet?

4. In Fig. 2, which is a scale drawing of a schoolroom, one-sixteenth of an inch represents 1 ft. How long is the room? The teacher's desk? How long are the pupils' desks? How wide are they?

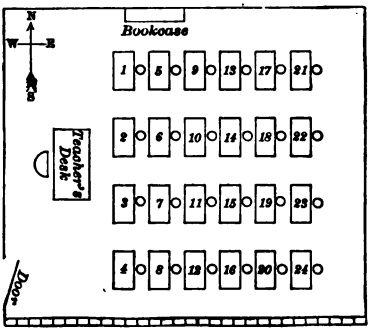


FIGURE 2

5. In what direction do the pupils face, when seated at their desks?

6. Find by measurement how far it is from the northwest corner of desk 1 to the southwest corner of desk 4; from the west edge of desk 2 to the east wall of the room.

7. Make and answer other questions on this drawing.

8. Make a similar (like) scale drawing from measurements of your schoolroom and the fixed objects in it.

9. Make a scale drawing from measurements of your schoolhouse and grounds.

§2. Reading Scale Drawings.

Using a foot rule marked to 8ths and 16ths of an inch and noticing the scale, answer by measurement the problems on the drawing of Fig. 1.

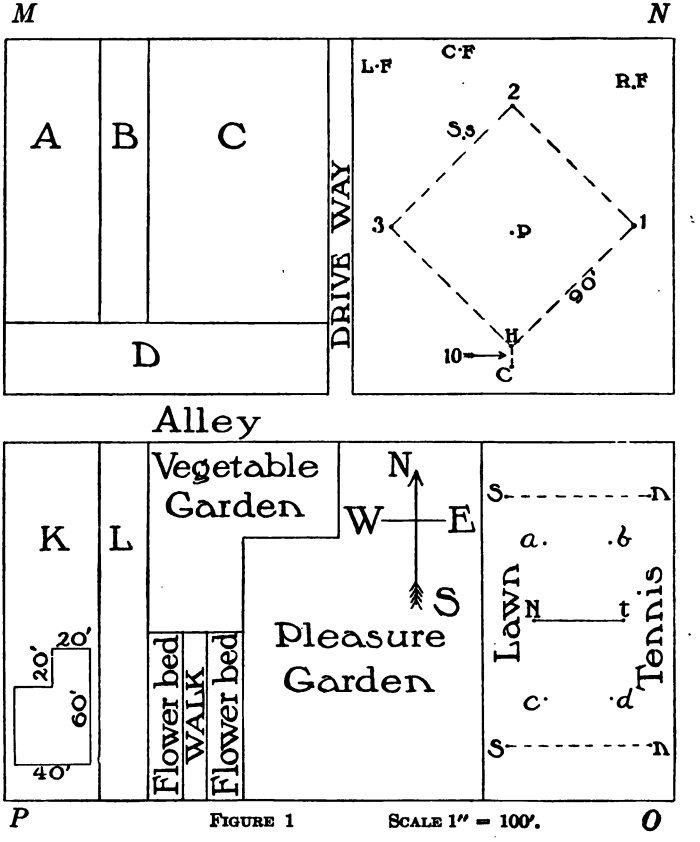


FIGURE 1 SCALE 1" = 100'.

1. How wide is the actual block that is shown by the drawing *M N O P?* How long is the block?

2. How wide is lot A? B? K? L?

3. How long is the vegetable garden? How wide?

4. How wide is the pleasure garden? The southeast corner lot? The northeast corner lot?

5. The alley (25 ft. wide) is taken equally from the north and south halves of the block. How long is lot K? The northeast lot? The southeast lot?

6. How long are the flower beds? How wide is the walk?

7. If a man cuts a strip along the west side of lot A with a lawn mower that cuts a swath 1 ft. wide, how many square feet of the lawn does he cut in passing along the side once? Twice? Ten times? How many strips must he cut to mow the entire lot?

8. How many square feet are there in lot A? B? In the northeast corner lot? In the southeast corner lot? In lot K?

9. The baseball "diamond" on the northeast lot is a 90-ft. square. How many square feet are there within the lines connecting the bases?

10. If a player strikes the ball and runs to third along the dotted lines, how far does he run? How far must a player run to make a "home run"?

11. The left fielder stands at L F and throws the ball to the pitcher at P. How far does he throw?

12. The center fielder at C F catches a ball which was struck at the home plate, throws it to third at 3, where it is caught and thrown to the home plate. How far has the ball gone?

13. How much will it cost to enclose the northeast quarter of the block with a fence costing 25 cents a foot?

14. Make and solve other problems on the drawing.

15. Notice the scale of any good maps in your geography and find by measurement about how far it is from Chicago to New York; to Washington, D. C.; to New Orleans; from New York to St. Louis; to Pittsburg; to Boston; to San Francisco.

NOTATION AND NUMERATION

§3. Digits. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

1. How many characters are given above? By what names do you know the first character? It is called *zero*, or *naught*.

These ten characters are called digits or figures.

By means of them all numbers are expressed.

§4. Place and Name Values of Digits.

1. Read the following numbers:

$30	3	30	30,000
300 mi.	.3	300	300,000
3 yd.	.03	3000	$\frac{3}{15}$

What is the same in all these numbers? What is different in the numbers?

2. Read:

7 feet 7 days 7 hundreds 7 millions 7 tenths

In all the numbers above, notice that the name of the *digit* is the same; but that the name of the unit of the *number* is different.

3. In the number 666, what does the first 6 on the right mean? The second 6? The third? *1st Ans.* ones *or* 6 units.

4. The value which depends on the name of a digit is called its *name value.* Suggest a name for the value which depends on its *place?*

5. How many values has a digit? What gives the first value? The second? Which is the permanent (unchangeable) value? Which is the changeable value?

§5. Reading Numbers.

1. Give the place value of 2 in each of the following numbers:

20 2 200 .2 20,000 2000 2222

2. In 286, for what number does the 2 stand? The 8? The 6?

We may read the number 2 hundreds, 8 tens, 6 units, or two hundred eighty-six, meaning two hundred eighty-six units.

Writing the names of the three places, they appear:

Hundreds	Tens	Units
2	8	6

meaning 200 + 80 + 6.

3. Read the following numbers, first as hundreds, tens, and units, and then as units:

729	916	335	494
845	642	538	739

4. Write the following in figures:

Forty-five thousand, two hundred eighty-four; six thousand, nine hundred thirty-four; three hundred twenty-one thousand, one hundred thirteen; five thousand, five.

NOTE.—Fill all vacant places with zeros.

5. Nine thousand, ten; ten thousand, nine; six hundred thousand, six hundred; five million, fifty thousand, three; eighty thousand, eighty.

§6. **Periods.** To aid in reading large numbers the digits are grouped into periods of three digits each.

The first three periods with their names and the names of the places in the periods are shown here:

Millions			Thousands			Units		
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
3	2	7,	5	8	6,	4	9	7

The number, 321,586,497, is read 321 million, 586 thousand, 497. The three-figure groups are read just as any three-figure number, then the name of the period is pronounced. After the last period, the units' period, the name of the period, that is the word "units," is omitted. The last period is "understood" to stand for units.

1. The table shows the number of men, women, children, and factories engaged in making pens and pencils, the wages, the cost of materials, and the value of the articles made in the United States during the years 1890, 1900, and 1905. Read some of the numbers:

	1890	1900	1905
Number of factories	41	55	62
Children employed	307	368	242
Women employed	1,001	1,562	2,287
Men employed	1,029	1,401	2,132
Wages of children	$46,913	$80,254	$48,149
Wages of women	$224,312	$393,289	$625,848
Wages of men	$570,699	$718,862	$1,122,999
Cost of materials	$1,211,850	$1,747,852	$3,072,980
Value of articles made	$3,025,664	$4,222,148	$7,673,777

2. Read the numbers of farm and work animals of the world for 1906, as given here:

Reindeer.... 654,588	Camels ... 2,803,595	Hogs 141,327,595
Donkeys ...8,221,072	Buffaloes .20,792,665	Cattle....424,499,238
Mules.....6,972,644	Horses ...91,486,403	Sheep....609,806,860

§7. Decimal Notation.

Answer all you can orally.

1. 10 equals how many units? 1 equals what part of 10?

2. 1 hundred equals how many tens? How many units? 1 ten equals what part of 1 hundred? 1 unit equals what part of 1 hundred?

3. 1 thousand equals how many hundreds? 1 hundred equals what part of 1 thousand?

4. The number shown by each digit in 1111 is how many times as great as the number shown by the digit to its right? The number shown by each digit equals what part of the number shown by the digit to its left?

5. How many tenths in 1 unit? 1 unit equals how many times 1 tenth (.1)?

6. 1 hundredth (.01) equals what part of 1 tenth? 1 tenth equals how many times 1 hundredth?

7. How does the place value of the unit change from right to left? From left to right?

8. In 432.5, what is the place value of the 4? Of the 3? Of the 2? Of the 5? The number is read: 432 *and* 5 tenths, that is, four hundred thirty-two *and* five tenths.

9. In 32.56, what is the place value of the 3? Of the 2? Of the 5? Of the 6? The number is read: 32 *and* 56 hundredths.

10. In 731.562, what is the place value of the 7? Of the 3? The 1? The 5? The 6? The 2? The number is read: 731 *and* 562 thousandths.

The period (.) is called the decimal point, or the point, because it points out the units' digit. It stands, or when not written, is assumed to stand, just to the right of the units' digit.

The part of the number to the right of the decimal point is called the decimal part of the number.

To read any number, as 689.308, that has a decimal part, read the part of the number on the left of the point (the 689) as a whole number. Call the point "and," then read the decimal part as though it were a whole number and, finally, pronounce the name of the last place on the right that is occupied by a figure.

Examples:

For 86.32, read: 86 and 32 hundredths.

For 479.841, read: four hundred seventy-nine and eight-hundred forty-one thousandths.

For 6802.329, read: 6802 and 329 thousandths.

For .682, read: 682 thousandths.

11. Read the following numbers:

8.7	175.25	8381.573	100.01
1.1	216.45	9260.235	170.17
10.1	640.78	10,632.106	200.02
23.4	3158.657	165,829.108	8000.008

12. Read these numbers distinctly:

700.07	321,000.321	6800.68
1600.16	147.147	101,000.101
1000.001	1200.12	10,700.107
47000.047	75,000.075	1100.011

§8. Writing Numbers.

Write in numerals the following:

1. Two hundred thirteen million, one hundred twenty-two thousand, six hundred seven; four hundred fifty million, three hundred twenty-seven thousand, one.

2. Two hundred million; one hundred ninety thousand, four hundred two; three million, one hundred twenty-four; fifteen thousand, nine; ninety; one hundred eighty-six million, ninety; one hundred forty-seven million, one hundred thousand, one hundred.

3. Eight tenths; eight hundredths; eighty hundredths; five and five tenths; six hundred and six hundredths; sixteen hundred and sixteen hundredths; twenty-five thousand and twenty-five thousandths; seven hundred twenty-eight thousand and seven hundred twenty-eight thousandths; three hundred eighty-six and three hundred eighty-six thousandths; fifty-six hundred and fifty-six hundredths.

The digits were first used by the Hindus. They were introduced into Europe by the Arabs, and we call the way of writing numbers with digits the Arabic notation.

Numbers written with these digits are called Arabic numerals.

§9. Roman Notation.

The Romans used letters instead of digits to represent numbers. This notation is still used upon monuments, and in numbering chapters and volumes of books and is called the Roman notation.

In the Roman notation,

I = 1	X = 10	D = 500
V = 5	L = 50	M = 1000
	C = 100	

When a letter of less value, as I, is placed before a letter of greater value, as V, the value of the less is to be taken from that of the greater; thus, IV means 4, and XC means 90.

When a letter of less value follows one of greater value, its value is added to that of the greater; as, VI for 6, XV for 15, and CX for 110.

Repeating a letter repeats its value; as, XX for 20, CC for 200.

A horizontal line over a letter increases its value a thousandfold; as, $\overline{C}$ meaning 100,000, $\overline{D}$ meaning 500,000.

To change to Arabic notation numbers written in the Roman notation, always begin on the right.

1. Change from Roman to Arabic numerals:

XV	MD	XIX	DDCV
XX	DM	XV	MDCCC
CCC	MCD	LIX	MCM
XC	MDC	LXI	$\overline{MD}$

2. Change from Arabic to Roman numerals:

20	69	800	505	1908
38	101	765	900	2000
60	305	648	1900	200,000

ADDITION

§10. Definitions.

Answer all you can orally.

1. A bicyclist rides 19 mi. Monday and 10 mi. Tuesday. How far does he ride both days?

2. I paid $28 for a suit of clothes and $20 for a bicycle. How much did I pay for both?

3. A four-sided lot has sides of the lengths, 25 yd., 4 yd., 11 yd., and 28 yd. How far is it around the lot?

4. A man works 7 hr. one day, 6 hr. the next day, 7 hr. the next, and 8 hr. the next. How many hours does he work in all?

5. A newsboy sold 18 papers Wednesday, 12 papers Thursday, and 16 papers Friday. How many papers did he sell in all?

6. The number of hours the sun shone on six of the days of a certain week was: 8 hr., 6 hr., 3 hr., 7 hr., 5 hr., 2 hr. How many hours of sunshine were there during the six days?

7. A farmer's fields are of the following sizes: 40 acres, 80 acres, 16 acres, and 34 acres. How many acres has he in all?

8. How many dollars are \$25, \$35, and \$20?

9. How many feet are 12 ft., 8 ft., 6 ft., 12 ft., and 4 feet?

The answer to problem 1 is the same as the answer to the question: "What single distance is just as long as the distances 19 mi. and 10 mi. combined?" Answering problem 2 answers the question: "What single amount equals \$28 and \$20 combined?"

Combining numbers into a single number is addition.

The result of addition is called the sum.

The numbers to be combined, or added, are the addends.

Thus, in the problem,

$$\left.\begin{matrix}19 \text{ mi.} \\ 10 \text{ mi.}\end{matrix}\right\} \text{Addends}$$
$$\overline{29} \text{ mi., Sum}$$

19 mi. and 10 mi. are the addends and 29 mi. is the sum.

The sign of addition is +. It is read "plus."

The sign, =, when written between two numbers means that they are equal. It should not be read "is," or "are," but "equal," or "equals."

10. Read and answer these questions:

(1) $3+8+4=?$ (4) $12+7+8=?$ (7) $3+15+4+9=?$
(2) $7+6+8=?$ (5) $6+13+7=?$ (8) $6+16+8+9=?$
(3) $9+8+5=?$ (6) $5+18+9=?$ (9) $5+14+9+7=?$

11. In these questions, what numbers should stand in place of the question mark?

(1) 2 8's+5 8's=? 8's. (4)* 7 x's+ 5 x's=? x's.
(2) 6 9's+7 9's=? 9's. (5) 9 y's+ 8 y's=? y's.
(3) 12 13's+7 13's=? 13's. (6) 15 z's+10 z's=? z's.

§11. Problems.

Use pencil and paper only when needed.

1. In five trips a street-car carried the following numbers of passengers: 30, 42, 45, 60, 65. Find the whole number of passengers carried.

When the sums can not be held in memory, it is convenient to arrange the addends in this form:

CONVENIENT FORM

30
42
45
60
65
12
23
242

SOLUTION

Writing the addends so that units, tens, hundreds, etc., shall stand in the same vertical column, add the units first; thus, 5, 10, *12* units = *1* ten and *2* units. Write the 2 units in units column under the line and the 1 ten in tens column. Then, 6, 12, 16, 20, *23* tens = *2* hundreds and *3* tens. Write the 3 in tens column and the 2 in hundreds column. Then add the partial sums as shown.

2. On these trips the conductor collected \$1.50, \$2.10, \$2.25, \$3.00, \$3.25. The fares for the five trips amounted to how much?

* Read (4) thus, "7 x's plus 5 x's equal how many x's?"

3. In one week a man's street-car fare was 35¢, 25¢, 40¢, 25¢, 30¢, and 30¢. Find the whole amount.

4. In one evening a telegraph operator sent messages containing the following numbers of words: 2, 6, 3, 12, 6, 3, 3. What was the total number of words dispatched?

5. Fred bought a note book for 10¢, a lead pencil for 5¢, a foot rule for 10¢, a compass for 25¢, and a bottle of ink for 10¢. What was the amount of his bill?

6. A train moves the following distances in five successive hours: 40 mi., 45 mi., 50 mi., 50 mi., 48 mi. What distance is traveled in the given time?

Find the total sales in each of the following problems:

7. Silk sales: $2\frac{1}{2}$ yd., $3\frac{1}{2}$ yd., $\frac{3}{4}$ yd., $2\frac{1}{4}$ yd., 3 yards.

8. Linen sales: $2\frac{1}{2}$ yd., 5 yd., 6 yd., $2\frac{1}{2}$ yd., 5 yards.

9. Ribbon sales: $1\frac{1}{2}$ yd., $\frac{1}{2}$ yd., 3 yd., 2 yd., 12 yards.

10. Thread sales: 25 spools white cotton, 6 spools black cotton, 3 spools black linen, 4 spools A silk, 6 spools twist.

11. Handkerchief sales: 6, 12, 3, 6, 12, 6, 3, 2. How many dozen were sold?

12. Notion sales: 6 papers pins, 2 papers safety pins, 6 papers needles, 2 cards darning wool, 2 combs, 12 bunches tape, 4 bunches braid. How many articles were sold?

13. Coffee sales: $\frac{1}{2}$ lb., 2 lb., $2\frac{1}{2}$ lb., 3 lb., $\frac{3}{4}$ lb., $1\frac{1}{4}$ lb., 5 lb., 4 pounds.

14. Sugar sales: 18 lb., 12 lb., 12 lb., 20 lb., 5 lb., 6 pounds.

15. Flour sales: 50 lb., 150 lb., 25 lb., 15 lb., 25 pounds.

16. Potato sales: 2 bu., $1\frac{1}{2}$ bu., 4 bu., 6 bu., 2 bu., $1\frac{1}{2}$ bu., 3 bushels.

17. Apple sales: 2 bu., 1 bu., 3 pk., 2 pk., 1 pk., 2 bu., $1\frac{1}{2}$ bu., 4 bushels. (Answer in bushels.)

18. Egg sales: 7 doz., 3 doz., 6 doz., 4 doz., 8 doz., 5 doz., 2 dozen.

19. Butter sales: 5 lb., 3 lb., 3 lb., $2\frac{1}{2}$ lb., $1\frac{1}{2}$ lb., 3 lb., 5 pounds.

§12. Applications.

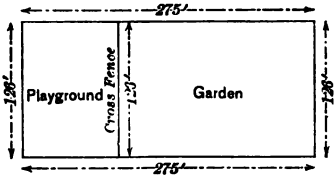


FIGURE 1

1. A fifth grade class staked out a rectangular plot of ground to be used for a playground and school garden. The eighth grade pupils fenced in the plot and ran a cross fence separating the parts. The entire plot was 126 ft. wide by 275 ft. long. How many feet of fence were needed?

CONVENIENT FORM

275 ft.
126 ft.
275 ft. } Addends
126 ft.
126 ft.
928 ft. Sum

SOLUTION:

Arrange the addends in convenient form. Beginning at the bottom of units column, think the sums thus, 12, 17, 23, *28* equals *2* tens and *8* units. Write the 8 in units column. Add the 2 tens to the tens digits, thinking (not speaking) 4, 6, 13, *22* tens, equals *2* hundreds and *2* tens. Write the 2 tens in tens place and add the hundreds to the hundreds in the third column, thus 3, 4, 6, 7, *9* hundreds. Write the 9 in hundreds place, and, since we are adding feet, the sum is 928 ft.

Test the correctness of the sum by beginning at the top of each column and adding downward. If the sum is the same the work is probably correct. This is called *checking* the work.

2. Add:

187, 892, and 478.
324, 407, and 282.
648, 376, and 197.

3. In a certain school there were enrolled one month:

96 children in the kindergarten	123 in the fourth grade
	100 in the fifth grade
182 in the first grade	95 in the sixth grade
143 in the second grade	83 in the seventh grade
133 in the third grade	76 in the eighth grade

How many pupils were in the whole school?

4. During the same month, the numbers of cases of tardiness were as follows:

Kindergarten..10	Third grade.. 8	Sixth grade...12
First grade....12	Fourth grade 5	Seventh grade..4
Second grade.. 4	Fifth grade.. 9	Eighth grade.. 2

What was the total number of cases of tardiness?

5. The numbers of cases of absence for the same month were as follows, beginning with the kindergarten: 87, 60, 46, 21, 15, 20, 15, 14, 9. Find the total number of cases

Teachers may take reports of their own or of other schools and make similar problems.

The length in feet, the number of officers and men, the displacement (the weight of water the vessel takes the place of, when standing still) in tons, and the horse power of 8 of the battleships of the United States navy are as given in the table below:

NAME	LENGTH	OFFICERS AND MEN	DISPLACEMENT	HORSE POWER
Alabama.....	368	585	11,525	11,366
Wisconsin.....	368	585	11,525	10,000
Kearsarge.....	368	520	11,525	11,954
Kentucky.....	368	520	11,525	12,318
Iowa.....	360	444	11,340	12,105
Indiana.....	348	465	10,288	9738
Massachusetts.....	348	424	10,288	10,403
Oregon.....	348	424	10,288	11,111
Total.....				

6. If these 8 vessels stood in a straight line with their ends touching, how far would they reach?

7. How many officers and men are needed to man the 8 ships?

8. What is the combined displacement of the vessels?

9. What is the combined horse power of the engines of the 8 ships?

10. Find the totals of the table below and tell what they mean.

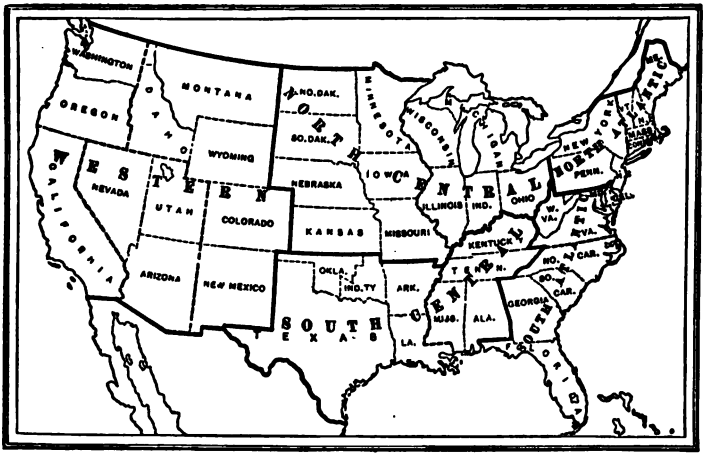
At the close of 1900 the numbers of teachers, schools, and pupils in the 6 provinces of Cuba were as follows:

Province	Teachers	Schools	Pupils
Havana	904	904	41,383
Puerta Principe	246	246	9,355
Santa Clara	879	879	44,872
Santiago	645	645	33,983
Pinar del Rio	274	274	13,282
Matanzas	619	619	29,398
Total			

§13. Distribution of Population in the United States in 1900.

On the map the main geographical divisions are surrounded by heavy full lines: ——. The divisions are separated into sections by heavy dotted lines: - - - -. Thus the New England states are the eastern section of the North Atlantic division; and New York, Pennsylvania, and New Jersey are the western.

Pupils may select such problems from this list as have a special geographical interest to their school.



Check the correctness of your work in the following problems, by adding columns downward, or in parts, and then adding the parts.

1. From the table of numbers given on the following page, find the total population of the North Atlantic division for 1900; for 1890.

2. Of the South Atlantic division for 1900; for 1890.

3. Of the North Central division for 1900; for 1890.

4. Of the South Central division for 1900; for 1890.

5. Of the Western division for 1900; for 1890.

6. Of the United States without outlying territory for 1900; for 1890.

7. Of the United States with outlying territory for 1900; for 1890.

8. Find the population of the Northern section of the South Atlantic division for 1900; for 1890.

9. Find the population of the Southern section of the South Atlantic division for 1900; for 1890.

POPULATION OF STATES AND TERRITORIES IN 1890 AND IN 1900

STATE, OR TERRITORY	POPULATION 1900	POPULATION 1890	STATE, OR TERRITORY	POPULATION 1900	POPULATION 1890
Me.....	694,666	661,086	Ky.....	2,147,174	1,848,635
N. H....	411,588	376,530	Tenn....	2,020,616	1,767,518
Vt	343,641	332,422	Ala.....	1,828,607	1,513,017
Mass	2,805,346	2,238,043	Miss . ..	1,551,270	1,289,600
R. I.....	428,556	345,506	La.....	1,381,625	1,118,587
Conn....	908,420	746,258	Tex.....	3,048,710	2,235,523
N. Y....	7,268,894	5,997,853	Okl.....	398,331	61,834
N. J.....	1,883,669	1,444,933	Ark.....	1,311,564	1,128,179
Penn....	6,302,115	5,258,014	Ind. T...	392,060	180,182
North Atlantic division ..			South Central division ..		
Del.....	184,735	168,493	Mont....	243,329	132,159
Md.....	1,188,044	1,042,390	Wy.....	92,531	60,705
D. C.....	278,718	230,392	Col.....	539,700	412,198
Va.....	1,854,184	1,655,980	N. M....	195,310	153,593
W. Va...	958,800	762,794	Ariz.....	122,931	59,620
N. C.....	1,893,810	1,617,947	Utah....	276,749	207,905
S. C.....	1,340,316	1,151,149	Nev.....	42,335	45,761
Ga.....	2,216,331	1,837,353	Idaho...	161,772	84,385
Fla.....	528,542	391,422	Wash. ..	518,103	349,390
			Ore.....	413,536	313,767
			Cal.....	1,485,053	1,208,130
South Atlantic division ..			Western division ..		
Ohio.....	4,157,545	3,672,316	U. S. without outlying territory..		
Ind.....	2,516,462	2,192,404			
Ill.....	4,821,550	3,826,351			
Mich.....	2,420,982	2,093,880			
Wis.....	2,069,042	1,686,880			
Minn....	1,751,394	1,301,826	Alaska..	63,592	32,052
Iowa....	2,231,853	1,911,896	Hawaii..	154,001	89,990
Mo.....	3,106,665	2,679,184	Phil. Is..	6,961,339	
N. D....	319,146	182,719	Tutuila..	6,100	
S. D.....	401,570	328,808	Guam...	9,100	
Neb.....	1,066,300	1,058,910	Porto Rico	953,243	
Kan.....	1,470,495	1,427,096			
North Central division ..			Total U. S. with outlying territory..		

10. Find the population of the eastern North Atlantic states for 1900 and for 1890; of the western North Atlantic states for both years.

11. Make and solve such problems as these:

Find the total population of the 13 original states for 1900; of the states east of the Mississippi river for 1900; for 1890, etc.

§14. Measurements.

A square unit, or a unit square, is a square each of whose sides is 1 unit long.

The area of a surface is the number of square units needed to cover it.

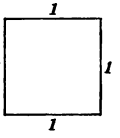


FIGURE 1

1. What is a square inch? A square foot? A square yard? A square mile? A square rod?

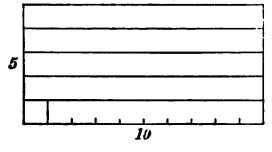


FIGURE 2

2. If a rectangle is made up of 5 rows of 10 square units each, what is its area? How many tens of square units are in the rectangle?

3. If a rectangle is 12 units long and 6 units wide, how many twelves of unit squares are there in the area?

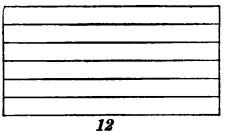


FIGURE 3

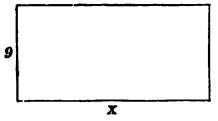


FIGURE 4

4. If a rectangle is x units long and 9 units wide, how many x's of unit squares are in the area?

NOTE.—Seven x's is written $7x$ and is read "seven x." It means 7 times x.

5. How wide must a rectangle z inches long be to contain $6z$ (6 times z) sq. in.? $15z$ square inches?

6. How long must a rectangle 15 in. wide be to contain $15a$ (15 times a) square inches?

7. The area of one rectangle is $8z$ sq. in. and of another $7z$ sq. in. How many square inches are there in their sum?

8. How many 9's are 8 9's and 7 9's? How many 12's are 8 12's and 7 12's? How many x's are $8x$ and $7x$? $8z + 7z = ?$

9. Add these numbers:

(1)	(2)	(3)	(4)	(5)	(6)
$9a$	$8a$	$26x$	$48z$	$76y$	$69b$
$8a$	$6a$	$47x$	$98z$	$49y$	$79b$

(7)	(8)	(9)	(10)	(11)	(12)
$6m$	$68n$	$84c$	$73d$	$786x$	$960x$
$8m$	$75n$	$76c$	$39d$	$319x$	$869x$
$9m$	$98n$	$37c$	$79d$	$876x$	$48x$

SUBTRACTION

§15. Definitions.

Answer all you can orally.

1. A man earns \$180 a month and spends \$140. What is the difference between his earnings and expenditures?

2. One farmer owns 320 A. of land; another 80 A. Find the difference in size of the two farms.

3. From a cheese weighing 43 lb., 23 lb. were sold. How many pounds remained?

4. From a bin containing 72 bu. of oats, I use 32 bu. How many bushels remain?

5. The sum of two numbers is 48, and one of the numbers is 28. What is the other?

6. What is required in the first two problems of this section? In the second two? In the fifth problem?

7. What is subtraction? What is the result in subtraction called?

Subtraction is the way of finding:

(1) The difference of two numbers expressed in the same units.

(2) The remainder after taking part of a number away from the number.

The number from which we subtract is the minuend. The number to be subtracted is the subtrahend. The result is the difference, or remainder.

Subtraction is also the way of finding either one of two addends when their sum and the other addend are known. The known sum is the minuend, the known addend is the subtrahend, and the unknown addend is the difference, or remainder.

The sign of subtraction, −, is read "minus."

When placed between two numbers the minus sign means that their difference is to be found.

The minuend is always written before the sign.

8. Tell what number the letter stands for in these problems:

(1) $8 - 5 = x$ (4) $25 - y = 8$

(2) $17 - 9 = x$ (5) $15 - 8 = z$

(3) $16 - 7 = y$ (6) $16 - z = 8$

9. What digit should take the place of the question mark in these problems?

(1) 7 8's − 5 8's = ? 8's (6) $9x - 6x = ?\ x$

(2) 8 12's − 3 12's = ? 12's (7) $15x - 7x = ?\ x$

(3) $4x - 2x = ?\ x$ (8) $15x - ?\ x = 8\ x$

(4) $5y - 3y = ?\ y$ (9) $18y - ?\ y = 9y$

(5) $9y - 5y = ?\ y$ (10) $17y - ?\ y = 8y$

Use pencil and paper only when needed.

1. A base-ball team won 58 games during the season, and lost 32. How many more games were won than lost?

2. A boy is 54 inches tall and his father is 72 inches tall. How much taller is the father than the boy?

3. A traveler has a journey of 437 mi. to make and he has already traveled 199 mi. of it. How much farther must he travel?

SOLUTION:

437 means 400 + 30 + 7, and 199 means 100 + 90 + 9. We arrange the numbers conveniently, thus:

Minuend	437 mi.	= 300 mi.	+ 120 mi.	+ 17 mi.
Subtrahend	199 mi.	= 100 mi.	+ 90 mi.	+ 9 mi.
Remainder	238 mi.	= 200 mi.	+ 30 mi.	+ 8 mi.

Beginning on the right, 9 units can not be taken from 7 units. We take one of the 3 tens and add it to the 7 units, giving 17 units. Then 17 units − 9 units = *8* units. Write *8* in units column. Passing to tens column, we can not take 9 tens from the 2 tens remaining, so we take one of the 4 hundreds and add it to 2 tens, making 12 tens. Then 12 tens − 9 tens = *3* tens. Write *3* in tens column. Finally, 3 hundreds − 1 hundred = *2* hundreds. Write the *2* in the hundreds column. The remainder is 238 miles.

4. Prove 238 to be the correct remainder, or difference, in problem 3 by adding it to the subtrahend, 199.

5. The minuend is 1284, and the subtrahend is 876. What is the difference, or remainder? Prove.

6. When the water was dried out of 38 oz. of natural soil, the dry soil weighed 29 oz. Make a problem from these facts, solve it, and prove your solution correct.

7. A subtrahend is 99, and the difference is 338. What is the minuend?

8. A subtrahend is 78, and the difference is 218. What is the minuend?

9. A minuend is 642, and the subtrahend is 167. What is the difference?

10. A subtrahend is 469, and the remainder is 299. What is the minuend?

11. From a wire 2 yards long is cut a piece 16 inches long. How long is the remainder?

12. From 72 hr. subtract 1 da. and 4 hours.

NOTE.—A correct answer to this problem would be 72 hr. − 1 da. and 4 hr. But the answer is to be expressed in a single unit, or name. To do this, first change 1 da. and 4 hr. to hours. 1 da. = 24 hr.; 24 hr. + 4 hr. = 28 hr. 72 hr. − 28 hr. = 44 hours.

13. From 36 ft. take 5 yd. What must first be done to obtain the result in a single unit?

14. 20 qt. $-$ 2 pk. $= x$ qt. What is the first thing to be done? Find the value of x.

15. From $\frac{3}{4}$ yd. take $\frac{3}{8}$ yd. What is the unit of $\frac{3}{4}$ yd.? Of $\frac{3}{8}$ yd.? What then must be done before subtracting? Why?

NOTE.—Only numbers expressed in the same unit can be subtracted if the result is to be expressed in a single unit.

§16. Exercises.

1. From a barrel of vinegar containing $31\frac{1}{2}$ gal., the following quantities were removed: 6 gal.; $2\frac{1}{2}$ gal.; 5 gal.; 3 gal.; $2\frac{1}{2}$ gal.; $2\frac{1}{2}$ gallons. Give the successive remainders.

2. From a wagon box containing 42 bu. of potatoes, a farmer sold the following quantities: 2 bu., 4 bu., 3 bu., 2 bu., 5 bu., 6 bu., 9 bu., 4 bu. How many bushels remained?

3. On March 31 the gas meter read 30,000 cu. ft., and on April 30, 45,000 cu. ft. How many cubic feet of gas had been used during the month?

4. A horse and his rider weigh 1375 lb. The man weighs 160 lb. What is the weight of the horse?

5. Find and read the differences in these exercises:

(1)	(2)	(3)
\$9274.75	\$4246.28	1,024,637 yd.
1846.19	1571.16	725,448 "

6. Find the differences:

(1)	(2)	(3)
6,420,014 rd.	\$3472.06	$1228\frac{3}{4}$
1,382,741 "	2189.24	$936\frac{1}{4}$

7. During one week a merchant's transactions (business dealings) at his bank were as follows: deposits (moneys put in), \$217.40, \$343.27, \$290.00, \$365.18, \$380.24, \$415.17; withdrawals (moneys taken out), \$200, \$320.25, \$195.75, \$286, \$240.15, \$395.25. Which item was the greater at the close of the week, and how much?

8. During one week of November, 1901, 63,188 cattle were received at the Chicago stock yards; during the corresponding week of 1899, 32,722 were received. What was the increase?

§17. Geography.

1. Great Salt Lake is 4200 ft. above sea level, and Lake Superior 602 ft. The difference in elevation is x. Find the number x stands for.

2. Cuba contains 45,884 sq. mi., and the Philippines, 114,410 sq. mi. Find the difference in size.

3. Lake Titicaca is 12,645 ft. above the level of the sea, and Lake Superior 602 ft. Find the difference in elevation.

4. Mt. Everest is 29,002 ft. above sea level, and Mt. Blanc 15,744 ft. What is the difference in altitude?

5. The Mississippi basin has an area of 1,250,000 sq. mi.; the Amazon, of 2,500,000 sq. mi. How much greater is the basin of the Amazon than that of the Mississippi?

6. The area of the Philippines is 114,410 sq. mi.; that of California, 158,360 sq. mi. How much larger is the state than the islands?

7. Compare the same islands with Texas, area 265,780 sq. mi.; with Illinois, area 56,650 sq. mi.; with Rhode Island, area 1250 square miles.

8. Of a population of 6,302,115 in Pennsylvania, 1,151,880 are pupils in elementary and secondary schools. How many persons are not attending those schools?

9. In a certain year the Hawaiian Islands produced 221,694 T. of sugar, while Cuba produced 880,372 T. How much more did Cuba produce than the Hawaiian Islands?

10. In 1890, 25,403 manufacturing establishments in New York City paid for wages $230,102,167; for materials, $366,422,722; for miscellaneous expenses, $59,991,710. The value of the products was $777,222,721. What was the total profit for all the establishments?

11. The battle of Lexington was fought in 1775, and the battle of Santiago in 1898. How many years elapsed between the two battles?

For the numbers for problems 12-18, see table, p. 130.

12. What is the difference of the populations of the North Atlantic states in 1890 and in 1900? Of the North Central division? Of the Western division?

Check the work in the following problems.

13. Which had the larger population in 1900, and by how much, the North Atlantic division or the South Atlantic division? In 1890? The north Central or the South Central division in 1900? In 1890? The South Central or the Western division in 1900? In 1890?

14. Compare the total populations of other groups of local interest.

15. Find, by comparing the populations for 1890 with those for 1900, the increase of population for the ten years in each state of the North Atlantic division; in the whole division.

16. Find the increase of population in your own state from 1890 to 1900. What is the average yearly increase?

17. Make and solve other problems, comparing the total populations of the different states; of the 13 original states; of the states east of the Mississippi river, or of divisions, in 1900 and in 1890.

18. Each principal division of states is separated into two or more sections by heavy dotted lines on the map, p. 129. Find how much greater, or less, the growth of population since 1890 has been in your section than in the other section, or sections, of the same division.

19. How many years since Illinois was admitted as a state? Since your native state was admitted into the Union?

20. Make similar problems from the table opposite, solve your problems, and check your work.

§18. Subtraction of Literal Numbers.

1. How many five-cent pieces are 7 five-cent pieces − 3 five-cent pieces?

2. How many dimes are 12 dimes − 5 dimes?

3. How many 9's are 16 9's − 9 9's?

4. How many c's are $13c - 7c$?

5. Write the differences:

$18x$	$26y$	$43z$	$68z$	$683a$	$1021z$
$9x$	$8y$	$37z$	$39z$	$97a$	$879z$

6. A lot contains $160x$ sq. ft., and the house covers $40x$ sq. ft. How many square feet of the lot are not covered by the house?

7. A boy earned $15x$ ¢. on Friday and spent $10x$ ¢. on Saturday. How many cents did he save?

DATES OF ADMISSION OF STATES AND TERRITORIES GEOGRAPHICALLY GROUPED

NORTH ATLANTIC DIVISION	DATE OF ADMISSION
Me	Mar. 3, 1820
N. H	June 21, 1788
Vt	Feb. 18, 1791
Mass	Feb. 6, 1788
R. I	May 29, 1790
Conn	Jan. 9, 1788
N. Y	July 26, 1788
N. J	Dec. 18, 1787
Penn	Dec. 12, 1787
SOUTH ATLANTIC DIVISION	
Del	Dec. 7, 1787
Md	April 28, 1788
D. C	July 16, 1790
Va	June 26, 1788
W. Va	Dec. 31, 1862
N. C	May 23, 1785
S. C	May 23, 1788
Ga	Jan. 2, 1788
Fla	Mar. 3, 1845
NORTH CENTRAL DIVISION	
Ohio	Nov. 29, 1802
Ind	Dec. 11, 1816
Ill	Dec. 3, 1818
Mich	Jan. 26, 1837
Wis	May 29, 1848
Minn	May 11, 1858
Iowa	Mar. 3, 1845
Mo	Mar. 2, 1821
N. D	Nov. 2, 1889
S. D	Nov. 2, 1889
Neb	Mar. 1, 1867
Kan	Jan. 29, 1861

SOUTH CENTRAL DIVISION	DATE OF ADMISSION
Ky	Feb. 4, 1792
Tenn	June 1, 1796
Ala	Dec. 14, 1819
Miss	Dec. 10, 1817
La	April 8, 1812
Tex	Dec. 29, 1845
Okla	Nov. 16, 1907
Ark	June 15, 1836
WESTERN DIVISION	
Mont	Nov. 8, 1889
Wy	July 11, 1890
Col	Aug. 1, 1876
N. M	Sept. 9, 1850
Ariz	Feb. 24, 1863
Utah	Jan. 4, 1896
Nev	Oct. 13, 1864
Idaho	July 3, 1890
Wash	Nov. 11, 1889
Ore	Feb. 14, 1859
Cal	Sept. 9, 1850
OUTLYING TERRITORY	
Alaska	July 27, 1868
Hawaii	April 30, 1900
Phil. Is	Nov. 28, 1898
Guam	Aug. 12, 1898
Porto Rico	Aug. 12, 1898
Tutuila	Feb. 16, 1900

8. Tell what number x stands for in these problems:

(1) $12-x=7$	(6) $7x-5x=10$	(11) $50x=100$
(2) $65-x=20$	(7) $16x-13x=15$	(12) $\frac{1}{2}x=50$
(3) $x-35=60$	(8) $9x-5x=20$	(13) $\frac{3}{4}x=27$
(4) $x-19=7$	(9) $7x=63$	(14) $\frac{2}{3}x=7$
(5) $3x-2x=8$	(10) $9x=108$	(15) $\frac{7}{8}x=63$

Literal numbers are numbers denoted by letters.

MULTIPLICATION

§19. Definitions.

Answer all you can orally.

1. At 75¢ a day, how many dollars does a boy earn in 6 days?

This problem may be solved in two ways. We may say he earned the sum of $75¢+75¢+75¢+75¢+75¢+75¢$, which is \$4.50.

Or, we may say he earned 6 times 75¢ ($6\times75¢$), or \$4.50.

In either case, his earnings are \$4.50.

2. Which is the shorter way? How do the addends compare in the first solution?

3. Find in two ways how far a vessel will sail in 4 da., making 22 mi. a day.

4. A factory employee, working by the hour, makes the following daily record for a week: 8 hr.; 8 hr.; $8\frac{1}{2}$ hr.; 9 hr.; 9 hr.; $9\frac{1}{2}$ hr. How many hours does he work during the week?

5. How can you solve this problem in more than one way? Give reason for your answer.

6. When the addends are unequal, as in problem 4, what is the only way they can be combined?

7. When the addends are equal, as in the first two problems, in how many ways can they be combined? Which is the shorter way? By what name is the shorter way known? What then is multiplication?

Multiplication of whole numbers is a short way of finding the sum of equal addends when the number of addends and one of them are given.

The given addend is the multiplicand.

The number of equal addends is the multiplier.

The result, or sum, is the product.

The multiplicand and multiplier are factors of the product.

The sign of multiplication, $\times$, is read "multiplied by" when it is written after the multiplicand and "times" when it is written before the multiplicand.

Thus, 22 mi. $\times$ 6 = 132 mi. is read "22 mi. multiplied by 6 equals 132 mi.," and 6 $\times$ 22 mi. = 132 mi. is read "6 times 22 mi. equals 132 miles."

An expression like 6 $\times$ 22 mi. = 132 mi. is called an equation.

8. Read and solve the following:

(1) 8 $\times$ \$75 = ? (5) 15 bbl. $\times$ 18 = ?

(2) 80 rd. $\times$ 7 = ? (6) 24 doz. $\times$ 10 = ?

(3) 11 $\times$ 64 ft. = ? (7) 45 bu. $\times$ 9 = ?

(4) 12 $\times$ 36 gal. = ? (8) 68 $\times$ 10 qt. = ?

9. At \$3.50 a pair, how much will 12 pairs of shoes cost?

\$3.50 = cost of 1 pair,

\$3.50 $\times$ 12 = cost of 12 pairs.

When a problem is expressed in an equation, the equation is called the *statement* of the problem.

§20. Problems in Multiplication.

Use pencil and paper only when needed.

Make statements and solve:

1. There are 128 cu. ft. in 1 cd. of wood. How many cubic feet in 15 cords?

STATEMENT. 15×128 cu. ft. $=?$

Or, 15×128 cu. ft. $= x$ cu. ft.

Find the number which should stand in place of x.

The number which should stand in place of x in an equation is called the *value of* x.

2. There are 5280 ft. in a mile. How many feet in 24 miles?

3. 12 lb. of ham at 17¢ per lb. costs how many dollars?

4. A cold wave from the northwest traveling at the rate of 32 mi. an hour moves how many miles in 24 hours?

5. If galvanized telegraph wire weighs 525 lb. per mi., how many pounds of wire will it take to stretch 6 wires from Chicago to Milwaukee, a distance of 82 miles?

6. How much will this wire cost at 8¢ per pound?

7. Find the cost of 275 fence posts at 65¢.

\$.65 = cost of one post;

275 = number of posts;

\$.65 × 275 = \$178.75 (cost of 275 posts) is the equation of the problem.

\$.65	In this problem, which number is the
275	multiplicand? Which the multiplier?
325	This problem may be solved
455	also thus: If each post cost 1¢,
130	275 posts would cost \$2.75; but
\$178.75	as each post costs 65¢, the cost of

\$2.75
65
1375
1650
\$178.75

all will be 65 × \$2.75, and the work may be arranged as shown. Or, we may say 275 times \$.65 is the same as 65 times \$2.75, and multiply as above.

Since the numerical product of factors is the same, whichever is used as the multiplier, the smaller number is generally taken for the multiplier *for convenience.*

8. In 64 pk. there are how many quarts?

9. How many pounds in 494 bu. of corn, if in 1 bu. there are 56 lb.? How many pounds in 25 bu.? How many in x bushels?

10. There are 231 cu. in. in 1 gallon. How many cu. in. in 587 gallons? In y gallons? In a gallons?

§21. Tables.

These products must be learned thoroughly:

$3 \times 3 = 9$	$3 \times 4 = 12 = 4 \times 3$
$4 \times 3 = 12 = 3 \times 4$	$4 \times 4 = 16$
$5 \times 3 = 15 = 3 \times 5$	$5 \times 4 = 20 = 4 \times 5$
$6 \times 3 = 18 = 3 \times 6$	$6 \times 4 = 24 = 4 \times 6$
$7 \times 3 = 21 = 3 \times 7$	$7 \times 4 = 28 = 4 \times 7$
$8 \times 3 = 24 = 3 \times 8$	$8 \times 4 = 32 = 4 \times 8$
$9 \times 3 = 27 = 3 \times 9$	$9 \times 4 = 36 = 4 \times 9$
$10 \times 3 = 30 = 3 \times 10$	$10 \times 4 = 40 = 4 \times 10$
$11 \times 3 = 33 = 3 \times 11$	$11 \times 4 = 44 = 4 \times 11$
$12 \times 3 = 36 = 3 \times 12$	$12 \times 4 = 48 = 4 \times 12$

Learn each of these in the same way:

$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$	$3 \times 9 = 27$
$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$
$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$	$5 \times 9 = 45$
$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$	$6 \times 9 = 54$
$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$	$7 \times 9 = 63$
$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$
$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$	$9 \times 9 = 81$
$10 \times 6 = 60$	$10 \times 7 = 70$	$10 \times 8 = 80$	$10 \times 9 = 90$
$11 \times 6 = 66$	$11 \times 7 = 77$	$11 \times 8 = 88$	$11 \times 9 = 99$
$12 \times 6 = 72$	$12 \times 7 = 84$	$12 \times 8 = 96$	$12 \times 9 = 108$

§22. Incomplete Problems.

Do as much of this work as you can orally.

At current prices, find the cost of the following articles:

1. $\frac{3}{4}$ lb. Oolong tea.
3 doz. eggs.
5 lb. ham.
3 lb. leaf lard.
2 lb. Java coffee.
3 lb. best quality of butter.

2. 2 erasers.
3 tablets linen paper.
3 pkg. white envelopes.
4 doz. pens.
2 lead pencils.
2 bottles writing fluid.

3. 2 doz. oranges.
2 doz. lemons.
$1\frac{1}{2}$ doz. bananas.
1 pk. apples.

4. 8 lb. rib roast.
$2\frac{1}{2}$ lb. porterhouse steak.
3 lb. lamb chops.
2 lb. sausage.

5. 3 cd. hard wood.
2 T. hard coal.
$1\frac{1}{2}$ cd. pine slabs.
4 T. soft coal.

6. 2 T. hay.
3 bu. oats.
2 bu. corn.
4 bales straw.

§23. Flooring and Lighting of Rooms.

Fig. 1 is a drawing, or plan, of the second story of a house. The scale is 1 inch to 16 feet, i.e. 1″ : 16′.

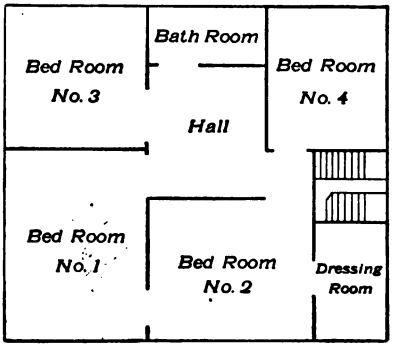


FIGURE 1

1. How long a distance in the house does a line of the drawing represent, that is 2 in. long? $1\frac{1}{2}$ in. long? $\frac{1}{2}$ inch long? $\frac{1}{4}$ inch? $\frac{1}{8}$ inch? $\frac{1}{16}$ inch?

2. Any line, or distance, in the drawing represents a line, or distance, how many times as long in the house?

NOTE.—The scale of the drawing might also be written 1:192.

3. With a foot-rule marked to 16ths of an inch, measure and give the width and length:

(1) Of the whole house
(2) Of bedroom No. 1
(3) Of bedroom No. 2
(4) Of bedroom No. 3
(5) Of the hall
(6) Of the bathroom
(7) Of the dressing room
(8) Of the space for stair steps

4. It costs 8¢ a square foot to floor the rooms with soft wood; 15¢ a sq. ft. to floor with hard maple; 24¢ a sq. ft. with oak, and 58¢ a sq. ft. with tile. What would it cost to floor each of the 7 rooms with soft-wood flooring? With hard-maple flooring? With oak flooring?

5. It was decided to put oak floors in the hall, in bedrooms No. 1 and No. 2, and in the dressing room; to put hard maple in bedroom No. 3; soft wood in bedroom No. 4, and tile in the bathroom. Find the total cost.

6. A room is said to be well lighted as a living room when the area of the window surface that lets in light is, at least, $\frac{1}{6}$ of the floor area of the room. Bedroom No. 1 is lighted by 3 windows, each containing 8 panes of glass, 16 inches by 20 inches. Is this room well lighted as a living room?

7. For a bedroom the total lighting surface of the windows need be only $\frac{1}{10}$ of the floor area. Is bedroom No. 1 lighted well enough for a bedroom?

8. Bedroom No. 2 is lighted by 2 windows, one containing 8 panes of glass 16 inches by 20 inches and one containing 12 panes of glass 16 by 20 inches. Is it well enough lighted for a bedroom? For a living room?

9. Bedroom No. 3 has 3 windows, 2 of them containing 8 panes of glass each, 16 by 20 inches, and the third contains 8 panes of glass 12 by 20 inches. Is it well lighted for a bedroom? For a living room?

10. My study is a room 12 ft. × 17 ft. and it is lighted by 3 windows, 2 of which contain 2 panes of glass each, 30 by 34 inches, and 1 of which contains 2 panes, each 30 by 40 inches. Is my study well lighted? Is it well lighted when curtains are half drawn?

11. Test whether your schoolroom is well lighted, supposing the total window surface that admits light should be at least $\frac{1}{5}$ of the floor area.

MISCELLANEOUS PROBLEMS

1. A double eagle weighs 516 grains, and a gold dollar 25.8 grains. Find the difference in weight between a double eagle and 2 gold dollars.

2. What will a peck of peanuts cost at 5¢ a pint?

3. Milk costs 6¢ a qt. and a family buys 2 qt. a day. What is their milk bill for April?

4. The average milk yield of a cow was 6 qt. a da. for 120 days. If this milk was all sold at $6\frac{1}{2}$¢ a qt., how much did the owner receive for it?

5. The cow's feed cost the owner $2.50 per mo. of 30 days. How much profit did the owner receive from the cow's milk during the 120 days?

6. A cow gives 12 lb. of milk a da. and the butter made from this milk equals $\frac{1}{30}$ of the weight of milk. How many pounds of butter does the milk furnish in 10 days?

7. If butter is selling for 35¢ a lb., what is the butter yield of this cow worth in 30 days?

8. 68 qt. of a certain Jersey cow's milk yield 7 lb. of butter. If milk is selling at 6¢ and butter at 35¢, is it more profitable to sell 68 qt. of the milk of this cow, or to make butter of it and sell the butter? How much more profitable is it?

9. The pulse of a healthy man beats 72 times per minute. The heart contracts once for each pulse beat. How many times do the muscles of a healthy man's heart contract in 1 hour? In 1 da. at the same rate?

10. Count your own pulse beats for 1 min. and find how many times your heart beats in a day at this rate.

11. Tie a heavy ball to a fine string, or split a bullet and fasten it to a thread, and suspend the string to a hook, or nail so that $1\frac{1}{2}$ ft. of it may vibrate (swing back and forth) freely. Start it swinging and count the number of vibrations per minute. At the same rate how many times would the ball vibrate in 24 hours?

12. Solve the same problem with the suspending string 3 ft. long; with the string 3 ft. 3 in. long.

13. My watch ticks 270 times per minute. How many times does it tick in 1 hour? In 1 day?

14. Light travels 186,600 mi. per second; how far does it travel in 1 day?

15. A man smokes 3 cigars a day, and pays 15¢ apiece for them. How much does this add to his expense account in 365 days?

16. A watch ticks 9 times in 2 seconds. How many times will it tick in 62 hours?

17. A boy reads 12 pages of history a day for $3\frac{1}{2}$ years. How many pages of history in all does he read?

§24. Multiplying by Factors.

1. $7 \times 8 = 56$ $28 \times 2 = 56$ $14 \times 4 = 56$

7, 8, 28, 2, 14, and 4 are all *factors* of 56.

2. Write the factors of 44, 48, 96, 35, 84, 81, 49, 108.

3. The two equal factors of 144 are 12 and 12. Give the two equal factors of 4, 9, 25, 49, 36, 121.

4.	$8 \times 2 = ?$	$8 \times 2 \times 5 = ?$	$8 \times 10 = ?$
5.	$12 \times 2 = ?$	$12 \times 2 \times 3 = ?$	$12 \times 6 = ?$
6.	$20 \times 4 = ?$	$20 \times 4 \times 3 = ?$	$20 \times 12 = ?$
7.	$25 \times 4 = ?$	$25 \times 4 \times 6 = ?$	$25 \times 24 = ?$
8.	$64 \times 9 = ?$	$64 \times 9 \times 7 = ?$	$64 \times 63 = ?$

9. Instead of multiplying a number by 15, by what two numbers in succession may I multiply it and get the same product?

10. Multiplying any number by 21 gives the same product as multiplying it by what two numbers in succession?

11. $56 \times 2 \times 4$ gives the same product as 56 multiplied by what single number?

12. Multiplying a number by several factors one after another gives the same product as multiplying it by what single number?

A number which has factors other than itself and 1 is a composite number.

A number such as 3, 5, 17, etc., which has no factors other than 1 and itself, is a prime number.

13. Instead of multiplying a number by a composite number, in what other way may I multiply it to obtain the same product?

14. Which of these numbers are prime, and which composite: 24, 25, 19, 6, 2, 21, 23, 84, 45, 43, 27, 28?

§25. Multiplying by 10, 100, 1000, 10,000.

1. Multiply 84 by 10; by 100; by 1000; by 10,000.

2. Multiply 327 by 10; by 100; by 1000; by 10,000.

3. How may any whole number be quickly multiplied by 10, 100, 1000, or 10,000?

§26. Multiplying by a Number Near 10, 100, 1000, 10,000.

1. $746 \times 9 = ?$

SOLUTION: This is once 746 less than 10×746, or $7460 - 746 = 6714$.

2. $965 \times 99 = ?$ *Ans.* $96{,}500 - 965 = 95{,}535$.

3. $965 \times 98 = ?$

4. $965 \times 101 = ?$ *Ans.* $96{,}500 + 965 = 97{,}465$.

5. $965 \times 102 = ?$

6. $873 \times 11 = ?$

7. $8473 \times 1001 = ?$

8. $8473 \times 999 = ?$

§27. Multiplying by 25, 50, 12½, 75, 500, or 250.

1. $65 \times 25 = ?$ $25 = \frac{1}{4}$ of 100; $\frac{1}{4}$ of $65 \times 100 = 1625$.

2. $83 \times 50 = ?$

3. $638 \times 12\frac{1}{2} = ?$

4. $132 \times 75 = ?$ $75 = \frac{3}{4}$ of 100; $\frac{3}{4}$ of $132 \times 100 = 9900$.

5. $640 \times 500 = ?$ $500 = \frac{1}{2}$ of 1000.

6. $740 \times 250 = ?$ $250 = \frac{1}{4}$ of 1000.

We see from problem 1 that to multiply by 25, we may multiply by 100 and take $\frac{1}{4}$ of the product.

7. Make a rule, different from the ordinary rule, for multiplying quickly by 50; by $12\frac{1}{2}$; by 250; by 500.

8. Make a rule for multiplying quickly by $33\frac{1}{3}$; by $333\frac{1}{3}$.

This shows the importance of remembering that:

$\frac{1}{2}$ of 100 = 50	$\frac{1}{16}$ of 100 = $6\frac{1}{4}$	$\frac{3}{8}$ of 1000 = 375
$\frac{1}{4}$ of 100 = 25	$\frac{1}{3}$ of 100 = $33\frac{1}{3}$	$\frac{5}{8}$ of 1000 = 625
$\frac{3}{4}$ of 100 = 75	$\frac{2}{3}$ of 100 = $66\frac{2}{3}$	$\frac{7}{8}$ of 1000 = 875
$\frac{1}{8}$ of 100 = $12\frac{1}{2}$	$\frac{1}{2}$ of 1000 = 500	$\frac{1}{16}$ of 1000 = $62\frac{1}{2}$
$\frac{3}{8}$ of 100 = $37\frac{1}{2}$	$\frac{1}{4}$ of 1000 = 250	$\frac{1}{3}$ of 1000 = $333\frac{1}{3}$
$\frac{5}{8}$ of 100 = $62\frac{1}{2}$	$\frac{3}{4}$ of 1000 = 750	$\frac{2}{3}$ of 1000 = $666\frac{2}{3}$
$\frac{7}{8}$ of 100 = $87\frac{1}{2}$	$\frac{1}{8}$ of 1000 = 125	

9. Make rules for multiplying quickly by the numbers on the right hand side of these equations.

§28. Multiplying by Fractional Numbers.

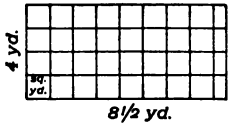


FIGURE 1

1. How many square yards in a room 4 yd. × $8\frac{1}{2}$ yards? (Here × is read "by.")

SOLUTION: How many sq. yd. fill a strip 1 yd. wide extending along the long side? How many such strips cover the floor? $8\frac{1}{2}$ sq. yd. × 4 = x sq. yd. What is x?

2. What is the cost of $68\frac{1}{2}$ acres of land @ $80?

SOLUTION: $80 × 68 = $5440, and $\frac{1}{2}$ of $80 = $40. Then, 80 × $68\frac{1}{2}$ = $5480. Observe that $80 × $68\frac{1}{2}$ means 80 × 68 + $\frac{1}{2}$ of 80.

3. Tell what these problems mean and then solve them:

(1) $64 × $16\frac{1}{8}$ = ?	(5) 897 yd. × $19\frac{2}{3}$ = ?
(2) 5280 mi. × $14\frac{2}{3}$ = ?	(6) $5\frac{1}{2}$ sq. yd. × $5\frac{1}{2}$ = ?
(3) 160 A. × $12\frac{3}{4}$ = ?	(7) 24 ft. × $8\frac{3}{8}$ = ?
(4) 81 lb. × $25\frac{1}{3}$ = ?	(8) $5\frac{1}{2}$ sq. yd. × 6 = ?

NOTE.—Solve the last problem by drawing a square $5\frac{1}{2}$ units long and $5\frac{1}{2}$ units wide, and dividing it up into square units.

The problems just solved show that $\frac{3}{4} \times 8$ means $\frac{3}{4}$ of 8, or 3 of the 4 equal parts of 8.

4. Give the meaning of these problems and find the value of the letter in each:

(1) $\frac{2}{3} \times 9 = x$ (3) $\frac{7}{8} \times 56 = y$ (5) $\frac{7}{13} \times 117 = z$

(2) $\frac{6}{7} \times 21 = x$ (4) $\frac{11}{12} \times 108 = y$ (6) $\frac{13}{15} \times 75 = z$

§29. Suggestions for Problems.

Make and solve problems based on the following facts:

1. A steel rail weighs 64 lb. per yd. of length. The distance from Washington to Baltimore is 42 mi.; to Philadelphia, 138 mi.; to New York, 227 mi.; to Boston, 459 miles.

NOTE.—There are 1760 yd. in 1 mile.

2. There are double tracks between Washington and each of the cities mentioned.

3. The distance from New York to San Francisco is 3262 miles.

4. The distance in miles from Chicago to 14 railroad centers of the United States are given here:

Indianapolis	184	Rochester	605
St. Louis	283	Baltimore	801
Cincinnati	298	Washington	820
Cairo	364	New Orleans	912
St. Paul	410	New York	913
Omaha	490	Denver	1028
Buffalo	536	San Francisco	2349

5. Galvanized telegraph wire weighing 572 lb. per mi. is used for distances over 400 mi., and for distances under 400 mi., wire weighing 378 lb. per mi. is used.

6. 8 wires run between Chicago and New York.

7. Galvanized iron telegraph wire costs 6¢ per pound.

8. Sound travels in water at the rate of 4708 ft. per second; in air, 1130 ft. per second.

§30. Rainfall.

How long is a cubic inch (Fig. 1)? How wide? How high? What is a cubic foot? A cubic yard?

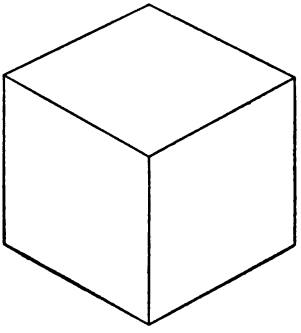


FIGURE 1

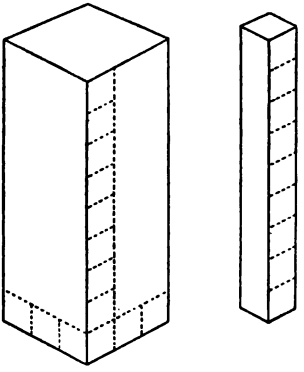


FIGURE 2

A tin box 3 in. square on the bottom and 9 in. high (Fig. 2) was used by a school as a rain-gauge. A second tin box 1 in. square on the bottom and 9 in. high was used to measure the depth of water in the large box. After the water was poured from the large box into the small box, the depth of water in the small box was measured with a thin stick and a foot-rule.

1. The rain-gauge was placed one evening where the rain could fall freely into its open top. During the night it rained and the next morning the water was poured from the gauge into the small box. It filled the small box to a depth of 9 in. How deep did the water fill the large box? How many cubic inches of water were caught in the large box?

One cu. in. of water for every sq. in. of surface is what is meant by 1 in. of rainfall.

2. How many cu. in. of water are there in a layer 1 in. deep in the large box (Fig. 1)? 2 in. deep? 5 in. deep?

3. How many cu. in. of water fell on 1 sq. ft. of the ground during a rainfall of 1 in. (Fig. 1)? Of 6 in.?

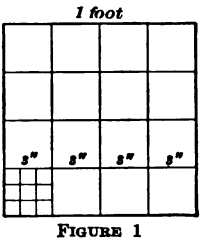


FIGURE 1

The number of cubic units (cu. in., cu. ft., cu. yd., etc.), a vessel holds, when full, is called its capacity.

4. What is the capacity of a square-cornered box $3'' \times 3'' \times 9''$?

5. What was the depth of rainfall during a shower if the large box caught enough water to fill the small box half full? To a depth of 3 in.? Of 6 in.? Of 1 in.? Of 2 in.? Of 7 inches?

6. How many cu. in. of water fell on 1 sq. ft. of the ground during a shower giving 1 in. of rainfall? 2 in.? $\frac{1}{2}$ inch?

7. During a rainfall of 1 in. how many sq. ft. of ground received enough water to make 1 cu. ft.? 2 cu. ft.? 12 cubic feet?

8. During June, 1902, the rainfall in the vicinity of Chicago was $6\frac{1}{2}$ in. During this month how many cu. in. of water fell on 1 sq. in. of ground? On 1 sq. ft.? On 12 sq. ft.? On 1 sq. yd.? On 30 sq. yd.? On $\frac{1}{4}$ sq. yd.? On $30\frac{1}{4}$ square yards?

9. Just before a shower set an uncovered bucket where the rain may fall freely into it. After the shower measure the depth of the water in the bucket to find the depth of rainfall. Find how many cu. in., or cu. ft. of water fell on each sq. ft.? Each sq. yd.? Each sq. rd. of the ground?

NOTE.—The bucket used must have the same size at the top and bottom, with straight sides. Why?

10. During the first 9 mo. of the year 1902, the region about Chicago received 32 in. of rainfall. How many cu. ft. of water fell during this time on a garden bed $10' \times 14'$? On a garden $48' \times 84'$? On a city block $250' \times 350'$?

11. If the walls of your schoolroom were water-tight, how many cu. ft. of water would it hold if it were filled 1 ft. deep? 2 ft. deep? 5 ft. deep? 8 ft. deep? To the ceiling?

§31. Problems with General Numbers.

$4 \times x$ is written $4x$. If $4x = 24$, what is the value of x?

Notice that the equation $4x = 24$ is the statement of this problem. A boy reads 24 pages of a book in 4 days. If he reads the same number of pages a day how many pages does he read a day? What does x stand for in this problem?

1. What number does x stand for in these equations?

(1) $3x=18$	(5) $x-18=4$	(9) $x+16=25$
(2) $6x=48$	(6) $x-9=8$	(10) $5x+2x=70$
(3) $8x=72$	(7) $x+9=17$	(11) $9x-3x=36$
(4) $x-8=15$	(8) $x+6=22$	(12) $8x+5x=26$

2. Write the sum of a and b.

3. Write the difference of a and b.

4. Write the product of a and b; of a and b and c; of x and y and z.

5. Answer the following questions if $a = 9$, $b = 8$ and $c = 6$:

$4a=?$ $12b=?$ $9c=?$ $ab=?$ $4ab=?$ $abc=?$ $3abc=?$

6. In the shortest way you can, write eight times x; seven times y; twenty-five times a times b; a times x times y.

7. In the shortest way, write and read a times b; c times x; fifteen times a times b times x.

DIVISION

§32. Division and Subtraction Compared.

Answer all you can orally.

1. A man owes a debt of $12 which he is to pay by work at $2 per day. How much will he owe at the end of the first day? Of the second day? Of the third day? Of the fourth day? The fifth? The sixth?

2. How long will it take to cancel the debt?

3. How many times may 2 be subtracted from 12, leaving no remainder? How many 2's are there in 12?

4. A man buys a horse for $90 and is to pay $15 a month until the horse is paid for. How much does the man owe after the first payment? After the second? The third? Fourth? Fifth? Sixth?

5. How many months will it take to pay for the horse?

6. How many times may 15 be subtracted from 90, leaving no remainder? How many 15's in 90?

7. What is one of the 6 equal parts of 12? Of 90?

8. A rectangular plot of ground 8 yd. wide by 18 yd. long is covered with bluegrass sod. How many square yards of sod does it contain? After a strip of sod 1 yd. wide, extending the length of the plot, has been removed, how many square yards of sod remain?

9. How many square yards remain after the removal of 2 such strips? Of 3? Of 4? Of 5? Of 6? Of 7? Of 8?

10. How many times may 18 sq. yd. be subtracted from 144 sq. yd., leaving no remainder? How many 18's are there in 144? What is one of the 8 equal parts of 144?

11. There are 160 bricks in a pile. Find by subtraction how many loads of 20 bricks each there are in the pile. How many 20's are there in 160?

Use pencil and paper only when needed.

12. Find by successive subtraction how many 88's there are in 616. What is one of the 7 equal parts of 616?

13. Find by subtraction how many months of 30 da. there are in 270 da. What is one of the 9 equal parts of of 270?

14. What number may be subtracted 4 times in succession from 120, leaving no remainder?

15. Find by subtraction how many times \$1263 is contained in \$5052.

16. Tell how to find by subtraction how many times 250 is contained in 1250.

17. Find in a shorter way how many 12's there are in 156.

18. Find in a shorter way than by subtraction one of the 14 equal parts of 224. By what name do you know this short way?

19. In the same way find one of the 15 equal parts of 225; of 210; of 240.

20. A lawn-mower cuts a strip 18 in. wide. How many strips must be cut to mow a lawn 18 ft. wide? 20 ft. wide? 30 ft. wide?

Division is a short way of subtracting one number from another a certain number of times in succession.

§33. Division and Multiplication Compared.

Answer all you can orally.

1. A horse traveled 72 mi. in 8 hr.; find the number of miles traveled per hour.

2. 4 bu. potatoes cost \$2.40. What was the price per bushel?

3. A room 4 yd. wide contains 24 sq. yd.; what is the length of one side?

4. 15 lb. sugar cost 90¢; what is the price per pound?

5. At \$6 per ton, how many tons of coal can be bought for \$180?

6. A floor containing 132 sq. ft. is 11 ft. wide; what is the length?

7. Find the cost of 10 pk. apples @ 14¢ per peck. If 10 pk. apples cost \$1.40, what was the price per peck?

8. How many pecks of apples @ 14¢ can be bought for \$1.40?

9. A train runs 420 mi. in 12 hr.; find the average number of miles per hour.

10. I paid 96¢ for veal at 12¢ per pound; how many pounds did I buy?

11. What is the cost of 3 T. hay at \$12 per ton?

12. At \$12 per ton, how many tons of hay can be bought for \$36?

13. A man paid \$36 for 3 T. hay; what was the price per ton?

14. Find the cost of 25 bbl. (barrels) flour at \$5.

15. Paid \$125 for 25 bbl. flour; what was the price per barrel?

16. Paid \$125 for flour at \$5 a barrel; how many barrels were bought?

17. The two factors of a number are 13 and 7; what is the number?

18. The product of two numbers is 91 and one of the numbers is 13; what is the other number?

19. The product of two numbers is 240 and one of the numbers is 20; what is the other?

20. $20 \times x = 240$; what number does x stand for?

21. Tell what number the letter stands for in each of these equations:

$12y=96$; $8x=56$; $7a=56$; $9b=72$; $10a=150$; $24m=240$.

Division is a way of finding one of two numbers when their product and the other number are given.

The product is called the dividend.

The given number is the divisor.

The required number is the quotient.

ILLUSTRATION.—The product of two factors is 490, and one of the factors is 7. What is the other factor?

$$7\overline{)490}\quad 70$$

We may also say that the dividend is the number to be divided, the divisor is the number by which the dividend is measured or divided, and the quotient is the measure.

The sign, ÷, of division is read "divided by," as 60 ÷ 12 = 5.

$60 \div 12 = 5$ may also be written in the following ways:

$12\overline{)60}\quad 5$ $\qquad$ $12)60(5$ $\qquad$ $\frac{60}{12}=5$ $\qquad$ $60/12=5$

$\qquad\quad 60$

The first and second are in common use in division.

In the third, the line placed between two numbers shows that the number above it is to be divided by the number below it; as in $\frac{1}{3}$, 1 is the dividend and 3 the divisor. The fraction itself is the quotient. The line between the two numbers is the division sign.

The fourth sign, /, is called the *solidus* (*sol'i-dus*).

§34. Short Division.

1. How many gallons are there in 296 pints?

SOLUTION: As there are 8 pt. in 1 gal. there are as many gallons in 296 pt. as there are 8's in 296.

$$8\overline{)296}\quad 37$$

2. How many days in 120 hours?

3. At \$8 per ton, how many tons of coal can be bought for \$240?

4. 984 marbles are distributed equally among a certain number of boys. Each boy has 82 marbles. How many boys are there?

5. Selling at 6 for a cent, how much will a dealer receive for 540 marbles?

6. A man paid \$2.60 for 4 bbl. of lime. What did one bbl. cost?

7. I bought 6 lb. of butter for \$1.62. What was the price per pound?

8. A dressmaker used 84 yd. of cloth for 6 dresses, allowing the same amount for each dress. How many yards were used in each dress?

9. A train ran 150 mi. in 6 hours. Not allowing for stops, what was the average number of miles per hour?

Find the value of x in problems 10 and 11:

10. 48 qt. $= x$ gallons.

11. At 6¢ a lb. I can buy x lb. of sugar for \$1.50.

12. 9 papers of needles cost 72¢. One paper costs how many cents?

13. In 1901, the number of trains entering Chicago every 24 hr. was about 1320. What was the average number per hour?

Answer all you can orally.

14. What does x stand for in each of the following equations?

(1) $63 \div 7 = x$	(4) $45 \div 3 = x$	(7) $120 \div 3 = x$
(2) $630 \div 9 = x$	(5) $96 \div 8 = x$	(8) $108 \div 9 = x$
(3) $48 \div 4 = x$	(6) $960 \div 8 = x$	(9) $72 \div 6 = x$

When dividend and divisor are small numbers, the quotient is readily seen. We say it is obtained by *inspection.*

When the divisor is a large number, the quotient is not readily found by inspection.

§35. Applications.

Use pencil and paper only when needed.

1. In 51,448 qt. how many pecks are there?

SOLUTION: In 51,448 qt. there are as many pecks as there are in 8 qt. in 51,448 quarts.

8 is contained in 51,000, 6000 times, with a remainder of 3000. Write 6 in the thousands place in the quotient. 3 thousands = 30 hundreds; 30 hundreds and 4 hundreds = 3400. 8 is contained in 3400, 400 times, with a remainder of 200. Write 4 in the hundreds place in the quotient. 200 = 20 tens; 20 tens and 4 tens = 24 tens. 8 is contained in 24 tens 3 tens times, without a remainder. Write 3 in the tens place in the quotient. 8 is contained in 8 units 1 unit time. Write 1 in the units place in the quotient.

$$8\overline{)51448}\quad 6431$$

In 51,448 qt, there are 6431 pecks.

Check: $6431 \times 8 = 51{,}448$.

2. There are 5280 ft. in a mile; how many yards are there in 1 mi.? In 2 miles?

3. If limestone weighs 160 lb. per cubic foot, how many cubic feet are there in a piece of limestone weighing 6400 pounds?

4. If marble weighs 170 lb. per cubic foot, find the number of cubic feet in a piece of marble weighing 5100 pounds.

5. If sand weighs 120 lb. per cubic foot, find the number of cubic feet in a load of sand weighing 4800 pounds.

6. A train of 12 sleeping cars is 840 ft. long. If the cars are all the same length, how long is each car?

7. A steel rail 30 ft. long weighs 720 lb.; what is its weight per yard of length?

8. An iron beam 24 ft. long weighs 1080 lb.; what is the weight of a piece of the beam 1 ft. long?

9. A steel girder (or beam) weighs 1728 lb. Each foot of length weighs 48 lb. How long is it?

10. There were 487,918 foreign immigrants to the United States in the year 1901. What was the average number per month? Per day? (30 da. = 1 month.)

11. During 1900 there were 448,572 immigrants. Find the average number per month.

12. Answer the same question for 1891, 1892, 1893, 1895, and 1897, the numbers for these years being in succession 560,319; 623,084; 502,917; 258,536, and 230,832.

13. In Albany, N. Y., there are 30 mi. of street railway, operated by 600 men. What is the average number of employees per mile?

14. From the data here given answer the same question for these cities:

City	Miles	Employees
St. Joseph, Mo	35	175
Memphis, Tenn	70	490
Oakland, Cal	80	560
Hartford, Conn	33	660
Worcester, Mass	43	453
Peoria, Ill	50	275

15. Find the value of x in the following:

(1) $3264 \div 6 = x$ (4) $89{,}765 \div 5 = x$ (7) $24{,}568 \div 8 = x$

(2) $9432 \div 12 = x$ (5) $78{,}870 \div 11 = x$ (8) $45{,}960 \div 12 = x$

(3) $2247 \div 7 = x$ (6) $75{,}699 \div 9 = x$ (9) $1{,}241{,}196 \div 11 = x$

16. How can you prove the correctness of your work in division?

17. Find what x equals in these equations:

(1) $\frac{80}{x}=10$ (3) $\frac{6300}{x}=9$ (5) $\frac{490}{x}=49$ (7) $\frac{640}{x}=20$

(2) $\frac{63}{x}=9$ (4) $\frac{5600}{x}=56$ (6) $\frac{810}{x}=90$ (8) $\frac{990}{x}=66$

§36. Long Division.

When dividend and divisor are both large numbers, it becomes necessary to show all the steps of the work.

1. 14,487 ÷ 33 =?

```
     9 }
    30 } =439, quotient
   400 }
33)14487
   13200   =400×33
   ----
    1287
     990   = 30×33
    ----
     297
     297   =  9×33
    ----
```

SOLUTION: Beginning at the left of the dividend; 33 is not contained in 1, nor in 14, but it is contained in 14,400, 400 times. Write the 400 above the dividend and subtract 400 × 33 = 13, 200 from the dividend, leaving 1287. This remainder must also be divided by 33.

33 is not contained a whole number of times in 1, nor in 12, but it is contained 30 times in 1280. Write the 30 above the 400 over the dividend and subtract 30 × 33 =990 from 1287, leaving 297.

33 is contained in 297, 9 times, leaving no remainder.

Thus we see 33 is contained in 14,487 400 + 30 + 9 = 439 times.

The work may be shortened a little by omitting the zeros and writing numbers in the shorter form.

SHORTER FORM

```
          439, quotient
divisor, 33)14487, dividend
            132
            ---
             128
              99
             ---
              297
              297
             ---
```

Check: 439×33 =14,487. Since this is the given dividend, the division is probably correct.

When a zero appears in the quotient proceed as follows:

2. 18,722 ÷ 46 =?

```
  407
46)18722
   184
   322
   322
   ---
```

SOLUTION: Begin as above. 46 is contained in 187, 4 times, with the remainder 3. Bring down the 2 in the dividend, giving 32. 46 is not contained in 32 a whole number of times. Write 0 in tens place in the quotient and bring down the next 2 of the dividend, giving 322. 46 is contained in 322, 7 times. Write the 7 in units place in the quotient.

Check: 407 × 46 = 18,722, which equals the dividend.

Complete these equations and check your work:

3. 3580 ÷ 45 =
4. 15,552 ÷ 64 =
5. 18,144 ÷ 56 =
6. 20,088 ÷ 72 =
7. 33,768 ÷ 72 =
8. 35,096 ÷ 82 =
9. 62,328 ÷ 84 =
10. 44,928 ÷ 96 =

§37. Miscellaneous Problems.

1. My friend is 1872 wk. old. How many years old is he?

2. There are 160 sq. rd. in 1 A. and a farmer pays 75¢ per acre for cutting and binding wheat. How much will it cost to cut and bind the wheat on a field 68 rd. by 80 rods?

3. A farmer paid a man $21.00 to shock his wheat, wages being $1.50 per day. How many days did the man work?

4. In 1880, 25 farm wagons sold for $2250 and in 1900, 15 such wagons sold for $855. How much less was the average cost of a farm wagon in 1900 than in 1880?

5. In 1880 a Minnesota farmer paid $3900 for 12 twine binders and in 1900, 14 twine binders cost him $1680. How much had the average price of twine binders fallen during these 20 years?

6. A steel rail weighing 72 lb. per yard is 30 ft. long.

How many men are needed to carry it, each man carrying 90 pounds?

7. In 25 da. a man earned \$56.25 husking corn at 3¢ per bushel. How many bushels per day did he husk?

8. A city lot 175 ft. long, containing 8750 sq. ft., sold for \$6000. If the short side fronts the street what was the price per foot of frontage?

9. The force required to draw a street car on a level track is 35 lb. per T. (2000 lb.) of the combined (total) weight of the car and its load. What force is needed to draw a car weighing 5600 lb. when it is loaded with 60 passengers whose average weight is 140 pounds?

10. At the speed of an ordinary horse car the pull of a horse equals about 125 lb. of force. How many horses will be needed to draw the loaded car of problem 9, the pull of no horse to be more than 125 pounds?

11. At a slow walk a horse can exert about 330 lb. of force. The force required to draw a loaded wagon on a level pavement is $\frac{1}{26}$ of the weight of the wagon and load. A coal wagon weighing 4860 lb. is loaded with 4 T. of coal. How many horses will be needed to draw the load over a level pavement, no horse drawing more than 330 pounds?

12. A horse can exert 1540 lb. of force for a few minutes. A box car weighing 30 T. is loaded with 32 T. The force needed to move the loaded car is $\frac{1}{86}$ of the combined weight of the car and load. How many horses will be needed to start the car on a level track?

§38. Geography.

1. The area of Massachusetts is 8315 sq. mi., and that of Illinois is 56,650 sq. mi. How many states the size of Massachusetts could be made from Illinois?

2. The area of New England is 66,465 sq. mi. How many states as large as New England could be made from Texas? (265,780 sq. mi.)

3. The area of Connecticut is 4990 sq. mi. and its population for 1900 was 908,420. How many persons per square mile are there in Connecticut?

NOTE.—In problems such as this, where the fractional part of the quotient has no meaning, drop the remainder if it is less than half the divisor, and add one unit to the whole part of the quotient if the remainder is more than half of the divisor.

4. The population of Connecticut for 1890 was 746,258. What was the population of Connecticut per square mile in 1890?

5. Answer questions 3 and 4 for your own state. (See table, p. 130).

6. From the same table answer questions 3 and 4 for Oklahoma territory. (area, 39,030 sq. mi.)

7. Make and solve similar problems for any states you are studying in your geography.

8. The area of Switzerland is 15,781 sq. mi. and its population is 2,933,334. What is the population of Switzerland per square mile?

9. 640 A. = 1 sq. mi. How many square miles in 534,528 acres?

10. The area of the state of Texas is 265,780 sq. mi.; that of New Jersey is 7815 sq. mi. How many states the size of New Jersey could be made from Texas? How many the size of Delaware, which contains 2050 square miles?

§39. Division by Multiples of 10.

Answer all you can orally.

1. Multiply each of these numbers by 10:

568 1268 306 $86.50 $8.65

2. Multiply each of the same numbers by 100; by 1000.

3. Divide each of these numbers mentally by 10:

5680 12680 3060 $865.00 $86.50

4. Make a rule for dividing any number quickly by 10; by 100; by 1000; by 1 with any number of zeros after it.

5. Name these quotients orally:

$60 \div 10 = ?$	$6 \div 2 = ?$	$60 \div 20 = ?$
$860 \div 10 = ?$	$86 \div 2 = ?$	$860 \div 20 = ?$
$\$64.20 \div 10 = ?$	$\$6.42 \div 2 = ?$	$\$64.20 \div 20 = ?$

Examining your answers to the questions just asked, make a rule for dividing a number quickly by 20; by 200; by 2000.

6.

$180 \div 10 = ?$	$18 \div 3 = ?$	$180 \div 30 = ?$
$630 \div 10 = ?$	$63 \div 3 = ?$	$630 \div 30 = ?$

From these answers make a rule for dividing any number quickly by 30; by 300; by 3000.

7. Make a rule for dividing a number quickly by 40; by 400; by 4000; by 50; by 800; by 1200; by 1500.

8. Make a rule for dividing any number quickly by any whole number of tens, as 40, 70, 90, 160; by any whole number of hundreds; of thousands.

9. Cutting off one zero from the right of a number has what effect on the number? Cutting off 2 zeros? 3 zeros?

$$165 \div 10 = 16\tfrac{5}{10} \text{ or } 16.5$$
$$75 \div 100 = \tfrac{75}{100} \text{ or } .75$$
$$478 \div 100 = 4\tfrac{78}{100} \text{ or } 4.78$$

10. Using first 10, and then 100, as a divisor show the quotients of the following:

400	500	2200	3300
460	790	4280	4860
287	439	9647	5732

11. There are 10 pk. in a barrel. How many barrels in 1488 pecks?

12. There are 60 lb. in a bushel of potatoes. How many bushels are there in 486 pounds?

13. 100 lb. = 1 cwt. How many hundredweight are there in 825 pounds?

14. 200 lb. pork = 1 bbl. How many barrels are there in 7624 pounds?

15. How many minutes in 4260 seconds?

SOLUTION: When there are ciphers at the right of both dividend and divisor, cut off an equal number of ciphers from both and divide.

$$60\overline{)4260}\quad 71$$

16. How many barrels will be needed for 4800 lb. of beef, allowing 200 lb. to the barrel?

§40. Other Methods of Shortening Division.

1. Solve these problems:

$$5000\overline{)25000} \qquad 500\overline{)2500} \qquad 50\overline{)250} \qquad 5\overline{)25}$$

How do the quotients compare? The dividends? The divisors?

2. Remembering that the dividends stand above the line and the divisors below, solve these problems:

$$\frac{324}{81}=? \qquad \frac{108}{27}=? \qquad \frac{36}{9}=? \qquad \frac{12}{3}=?$$

How do the quotients compare? The dividends? The divisors?

3. How do the quotients, the dividends, and the divisors compare in these problems:

$$\frac{256}{128}=? \qquad \frac{64}{32}=? \qquad \frac{16}{8}=? \qquad \frac{4}{2}=?$$

4. To divide 324 by 81 what smaller numbers may be

used to get the same quotient? How may these smaller numbers be obtained from 324 and 81?

SOLUTION: We see that, as $324 = 27 \times 12$ and $81 = 27 \times 3$, we may write

$$\frac{12 \times 27}{3 \times 27} = \frac{12}{3} = 4.$$

This can be indicated thus:

$$\frac{12 \times \cancel{27}}{3 \times \cancel{27}} = 4.$$

Removing these factors is called cancellation. It can often be used to simplify the division of products.

5. Answer the same questions for $256 \div 144$.

These problems show that any factor of both dividend and divisor may be dropped from both and the remaining factors divided.

6. Solve these problems by cancellation:

(1) $\frac{34 \times 16}{62 \times 1} = ?$ (2) $\frac{6 \times 8 \times 3}{2 \times 8 \times 3} = ?$ (3) $\frac{15 \times 21 \times 846}{3 \times 7 \times 846} = ?$

(4) $\frac{18 \times 3 \times 4 \times 67}{2 \times 4 \times 3 \times 67} = ?$ (5) $\frac{625}{25} = ?$ (6) $\frac{324}{189} = ?$ (7) $\frac{85}{34} = ?$

To use cancellation effectively, methods of finding factors of numbers are necessary.

§41. Tests of Divisibility.

1. Which of these numbers are exactly divisible (can be exactly divided) by 2?

12 24 36 23 45 18 37 40 59 61

What are the last digits of the numbers which 2 will exactly divide?

TEST FOR THE FACTOR 2.

If a number ends in 0, 2, 4, 6, or 8, it can be exactly divided by 2.

2. Which of the numbers, 24, 30, 45, 50, 700, 640, 83, 765, 6400, are exactly divisible by 10?

What is the last digit of the numbers which 10 exactly divides?

TEST FOR THE FACTOR 10.

If a number ends in 0, 10 exactly divides it.

3. Make a rule for testing whether 100 divides a number.

4. Which of the numbers, 16, 18, 35, 25, 60, 20, 28, 65, 460, 672, 1260, does 5 exactly divide?

What are the last digits of the numbers which 5 will exactly divide?

TEST FOR THE FACTOR 5.

If a number ends in 0 or 5, 5 exactly divides it.

5. Which of the numbers, 12, 17, 24, 81, 27, 93, 64, 75, 126, 324, 185, does 3 exactly divide?

Of those which 3 exactly divides, will 3 also exactly divide the sum of the digits? Of the numbers 3 does not exactly divide, is the sum of the digits exactly divisible by 3?

TEST FOR THE FACTOR 3.

If 3 exactly divides the sum of the digits of a number it divides the number also.

6. Of the numbers 126, 368, 453, 729, 819, 639, 2358, what ones does 9 exactly divide?

See whether 9 will exactly divide the sum of the digits of the numbers that 9 exactly divides.

TEST FOR THE FACTOR 9.

If 9 exactly divides the sum of the digits of a number it also exactly divides the number.

7. Which of the numbers, 113, 124, 368, 560, 375, 486, 1204, does 4 exactly divide:

If 4 exactly divides a number, does it also exactly divide the number indicated by the last two digits?

For example, 4 is a factor of 368 and also of 68.

TEST FOR THE FACTOR 4.

If the number denoted by the last two digits of a number can be divided by 4, the entire number can be exactly divided by 4.

8. If 8 exactly divides a number, does it also exactly divide the number indicated by the last three digits?

TEST FOR THE FACTOR 8.

If the number denoted by the last three digits of a number can be divided by 8, the entire number can be exactly divided by 8.

9. Which of the numbers, 60, 175, 285, 625, 1350, 1275, 8645, 8675, 8625, 8650, are divisible by 25?

25 divides 175 exactly and 25 also exactly divides 75, which is the number denoted by its last two digits. Is this true of all numbers 25 exactly divides? Is it true of any numbers 25 does not exactly divide?

TEST FOR THE FACTOR 25.

If the number denoted by the last two digits of any number is divisible by 25 the entire number is divisible by 25.

TEST FOR THE FACTOR 6. Test for both 2 and 3.

TEST FOR ANY COMPOSITE FACTOR. Test singly for all the factors of the composite factor.

10. TEST for divisibility by such numbers as 36, 216, 27, 49, etc., which contain some factor two or more times.

11. Pick out the numbers of this list which are exactly divisible by 2:

6 81 65 72 86 129 9864 8643 7986 16,835 29,860

12. Pick out those which can be exactly divided by 3; by 9; by 4; by 8; by 5; by 10; by 12; by 15.

§42. Checking Division.

Division may be checked, or tested, by multiplying the divisor by the quotient and adding the remainder

to the product. If the sum equals the dividend, the work is checked.

Another check is to divide by the factors of the divisor successively, and note whether the final quotient is the same as that given by the complete divisor.

§43. Applications of Cancellation.

In all the problems of this list indicate your divisions and multiplications, then apply the tests of divisibility, and cancel the factors found in both dividend and divisor. Multiply the uncancelled factors in the dividend together and divide this product by the product of the uncancelled factors of the divisor.

ILLUSTRATION.—A speed of 102 mi. per hour equals how many feet per second? Indicate the work thus:

$$\frac{\overset{\overset{44}{\cancel{440}}}{\cancel{102}\times\cancel{5280}}}{\underset{\underset{1}{\cancel{10}}}{\cancel{60}\times\cancel{60}}} = \frac{17\times 44}{5} = \frac{748}{5} = 149\tfrac{3}{5}.$$ *Ans.* $149\frac{3}{5}$ ft. per second.

1. It took 225,280 lb. of steel rails to lay 1 mi. of single-track railroad. What was the average weight of the rails per yard for the track?

	LB. PER CU. FT.	CU. FT. PER T.
Granite.....	170	
Limestone	160	
Marble.....	170	
Sandstone	140	
Slate.....	170	
Cast Iron.....	450	
Steel.....	480	

2. Find the number of cubic feet in a ton of 2000 lb. for each substance given in the table.

3. The average speed of American express trains is about 35 mi. per hour for long distances. Find the speed per second.

SOLUTION: Put work in this form:

$$\frac{\overset{7}{\cancel{35}}\times\overset{22}{\cancel{5280}}}{\underset{1}{\cancel{60}}\times\underset{1}{\cancel{60}}} = \frac{154}{3} = 51\tfrac{1}{3}. \quad \textit{Ans.}\ 51\tfrac{1}{3}\ \text{ft. per second.}$$

4. In both England and America, the average speed of express trains for distances from 100 to 250 mi. is about 40 mi. per hour. How many feet per second is this?

5. For long distances the average speed of English express trains is about 43 mi. per hour. How many feet per second is this?

6. In 1893 an express train in the United States ran 1 mi. at the rate of 98 mi. per hour. How many feet per second is this?

7. A railroad train ran for 1 min. at the speed of 130 mi. per hour. Find the number of feet per second which it traveled.

8. Sound travels in air at the rate of about 750 mi. per hour. How many feet per second is this?

9. Cannon balls have been thrown at a speed of 810 mi. per hour for a few seconds. This is how many feet per second?

10. The moon moves around the earth at a speed of about 1,840,000 mi. in 30 da. How many feet is this per second?

11. In 365 da. the earth moves around the sun through a distance of about 584,000,000 mi. What is the earth's speed in miles per second?

12. The earth, by turning on its axis, carries a place on its equator about 25,000 mi. in 24 hours. How many feet is it carried per second?

13. A farmer bought 18 bu. of wheat at 75¢ and paid the bill with potatoes at 50¢ per bushel. How many bushels of potatoes were required?

14. A farmer bought 80 acres of land at $100 per acre and paid for it with land worth $60 per acre. How many acres were required?

15. A man buys 5 acres of city land at $720 per acre and pays for it with 40 acres of farm land. What price per acre does the farm land bring?

16. How many bricks each $2'' \times 4'' \times 8''$ are there in a straight pile $12'' \times 36'' \times 20'$? In a straight pile $2' \times 6' \times 12'$?

17. How many lots each $25' \times 150'$ can be made from a town block $250' \times 300'$?

§44. Farm Products of the United States in 1900.

In these problems use the tests for divisibility to discover common factors in divisor and dividend, and then remove these factors by cancellation. The numbers that remain in dividend and divisor are then to be divided in the usual way. Omit fractions in the quotients.

1. There were 512 coffee farms in the United States in 1900, having a total area of about 70,200 acres. Find the average size of a coffee farm.

SUGGESTION.—Test for the factor 8 in dividend and divisor.

2. The total value of these 512 coffee farms was about $1,933,000. What was the average value of a farm?

3. If the total acreage (number of acres) of the coffee farms was about 70,200, and their total value about $1,933,000, what was the average value of an acre of coffee land in the United States?

SUGGESTION.—Test for the factor 100 and then for 2.

4. There were 441 farms in the United States devoted to raising a certain food plant, having a total acreage of about 18,900. Find the average size of one of these farms.

SUGGESTION.—Test first for 9, then remove the common factor 7 from both dividend and divisor.

5. The total value of these 441 farms was about $562,500. Find the average value of one of these farms.

6. If the total acreage in these farms was 18,900 and the total value $562,500, find the average value of an acre for raising this plant.

7. There were in our country about 5720 rice farms, having a total acreage of about 108,800, and a total value of about $17,834,000. Find the average acreage, and the average value of a rice farm, also the average value of an acre for raising rice.

8. About 2025 nursery farms had a total acreage of about 165,800, and a total value of $19,146,000. What was the average acreage, the average value of a nursery farm, and the average value of land per acre for nursery purposes?

9. For sugar farms these figures were nearly correct: 7340 farms, of 2,669,000 acres, worth $150,420,000. Solve problems like 7 and 8 for sugar farms.

10. The numbers for farms devoted to raising flowers and plants were about 6160 farms, of 42,660 acres, worth $52,460,000. Solve problems like 7 and 8 for flower- and plant-raising farms.

11. From these numbers, solve like problems for fruit farms: number of farms 82,200; total acreage, 6,150,000; total value, $440,000,000.

12. Further problems may be made on this table, which is a fairly complete list of the farm products of the country.

Product	Number of Farms	Acres in Farms		Value of Farms	
		Total	Average	Total	Average
Tobacco.....	106,000	9,570,000.....		$ 215,500,000	.....
Vegetables.....	186,000	10,160,000.....		547,000,000.	.....
Dairy produce..	358,000	43,300,000.....		1,693,500,000	.....
Cotton.....	1,072,000	89,600,000.....		1,108,000,000	.....
Hay and grain.	1,320,000	210,260,000.....		6,380,000,000	.....
Live stock.....	1,565,000	355,000,000.....		7,505,300,000	.....
Miscellaneous..	1,060,000	113,200,000.....		2,383,700,000	.....

§45. The Thermometer.

The official record of hourly temperatures for Chicago from 7 P.M. February 27 to 7 P.M. March 1, 1902, as given in a daily paper, was as follows:

Feb. 27; Night	Feb. 28; Day	Feb. 28; Night	March 1; Day
7 P.M.....38°	7 A.M.....41°	7 P.M.....38°	7 A.M.....34°
8 P.M.....37	8 A.M.....41	8 P.M.....37	8 A.M.....33
9 P.M.....37	9 A.M.....41	9 P.M.....37	9 A.M.....33
10 P.M.....39	10 A.M.....43	10 P.M.....37	10 A.M.....32
11 P.M.....39	11 A.M.....44	11 P.M.....37	11 A.M.....32
12 midnight.38	12 M.....38	12 midnight.36	12 M.....32
Average....	Average	1 A.M.....36	1 P.M.....31
1 A.M.....38	1 P.M.....35	2 A.M.....35	2 P.M.....31
2 A.M.....38	2 P.M.....35	3 A.M.....34	3 P.M.....30
3 A.M.....38	3 P.M.....37	4 A.M.....34	4 P.M.....29
4 A.M.....37	4 P.M.....39	5 A.M.....34	5 P.M.....28
5 A.M.....39	5 P.M.....40	6 A.M.....34	6 P.M.....28
6 A.M.....40	6 P.M.....39	Sum.....	Sum.....
Average	Average	Average	Average ....

Note.—The *average* of temperatures, or numbers, is found by adding them and dividing the sum by the number of temperatures, or numbers, which have been added. *To average* means to find the average.

1. Average the temperatures on Feb. 27 from 7 P.M. to midnight. From midnight to 6 A.M.

2. What was the average temperature on Feb. 28, from 7 A.M. to 12 M.? To 7 P.M.?

3. What is the difference between the two averages of problem 1? Of problem 2?

4. What is the difference between the lowest and highest temperatures from 7 P.M., Feb. 27 to 7 P.M., Feb. 28? This is called the *range* of temperature for the day.

5. The average temperature for Feb., for 30 years, is 26°. How much does the average temperature for the 24 hr., following 7 P.M. Feb. 27, exceed the thirty-year average?

6. Find the average temperature for the night of Feb. 28. For the day of March 1.

7. Find the difference between the seaverages. What does this difference show?

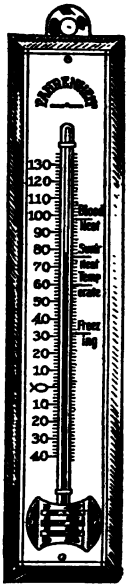
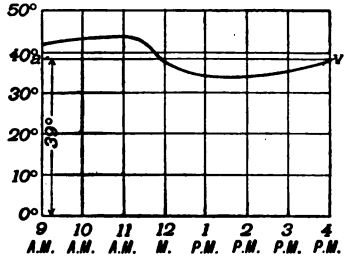


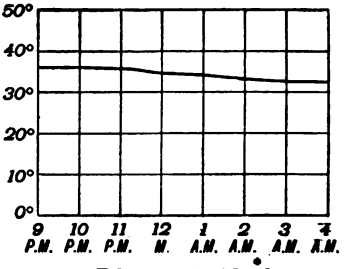
FIGURE 1

8. Make and solve such problems from outdoor thermometer readings at your schoolhouse.

9. What is the reading of the thermometer of Fig. 1? What would the thermometer read if the top of the mercury column stood at "freezing"? At "summer heat"? At "blood heat"?



February 28
FIGURE 2



February 28—*March* 1
FIGURE 3

10. Lay off on cross-lined paper the readings as the readings of the table from 9 A.M. to 4 P.M. of Feb. 28 are laid off in Fig. 2; draw a smooth curve, free-hand, through the points. Such a curve gives a picture of temperature changes.

11. From a daily paper obtain the hourly readings from 9 P.M. to 4 A.M., and lay them off to scale as the readings of the table from 9 P.M., Feb. 28, to 4 A.M., March 1, are laid off in Fig. 3. Draw the free-hand curve as before and find whether the day or night temperatures are the *steadier* (more regular).

12. Average your readings from 9 A.M. to 4 P.M., and draw a straight, horizontal line on your diagram, at a distance above the horizontal zero line equal to the average, as the line, *av*, of Fig. 2 is drawn. This is the *average line* for these 8 hours.

§46. Exercises for Practice.

Check results in division by multiplying divisor by quotient and adding the remainder if there is one.

SHORT AND LONG DIVISION

1. Divide 1260 by 2; by 3; by 6; by 7; by 8.
2. Divide 2880 by 3; by 4; by 5; by 6; by 9.
3. Divide 3402 by 3; by 7; by 21; by 4; by 12.
4. Divide 2520 by 4; by 5; by 20; by 7; by 30.
5. Divide 33264 by 6; by 7; by 8; by 42; by 56.
6. Divide 50400 by 10; by 30; by 40; by 50; by 60.
7. Divide 201600 by 200; by 300; by 400; by 500.
8. Divide 3360 by 7; by 8; by 56.
9. Divide 2880 by 8; by 9; by 72.
10. Divide 57672 by 6; by 2; by 12; by 72.

11. How may you divide a number by 35 by short division? By 56? By 63? By 80? By 72? By 54? By 48? By 84?

LONG DIVISION

12. Divide 360360 by 120; by 210; by 336; by 416.

13. Divide 5045040 by 42; by 105; by 315; by 540; by 702; by 862; by 1404.

14. Divide 6387624 by 81; by 320; by 576; by 736.

15. Divide 8736824 by 632; by 8261; by 42125.

16. Divide 30200610 by 700; by 1764; by 1908.

17. Divide 23642901 by 21; by 86, by 364; by 4081; by 19881; by 98631.

18. Divide 38083584 by 672; by 816; by 2688; by 8064.

BILLS AND ACCOUNTS

§47. Exercises.

	Mon.	Tu.	Wed.	Th.	Fri.	Sat.	Wages	
	hours	hours	hours	hours	hours	hours	$	¢
Adams.....	7½	8	8	8½	9	8¾		
Benson	8	8½	8¼	8¾	8½	9		
Boyd.....	0	4	6	6	6½	7		
Claussen...	8	8½	8½	8½	9	8½		
Denning....	8½	9	8½	8¼	9	9		
Doan.....	8	8	8	8	8	8		

1. Find the number of hours each man worked.

2. Find the number of hours all the men worked.

3. At 30¢ an hour, find the daily wages of each man.

4. Find the amount due each man at the close of the week.

5. Find the amount due to the six men at the close of the week.

6. Which man worked the greatest number of hours? The smallest number?

7. Counting 9 hr. to the day, what is the full amount any one man would earn in a week?

8. Counting 8 hr. to the day, how much "overtime" did each man work?

9. For how many extra hours did five of the men work, counting 8 hours to the day?

10. If double wages were paid for "overtime," how much did each man earn during the week? How much did all earn?

11. How many hours did the third man lose?

§48. Problems Partly Original.

1. Adams paid \$4 on a doctor's bill, and put aside \$2 for rent.

2. Benson has a wife and two children. Average the amount received for each one in the family.

3. How much does Boyd's loss of time reduce his wages for the week?

4. Claussen paid \$10 for clothing, and \$3.50 for the week's board.

5. Denning saves \$3 a week to pay the premium (yearly charge) on a life insurance policy; pays \$3.50 a week for board, and 35¢ for laundry.

§49. Accounts.

A man has on hand at the beginning of February, 1902, \$250. He receives during the month \$200 for salary and \$60 for the rent of two houses. During the month he

pays $72.50 on an insurance policy; $110 for household expenses; $25.50 for incidentals, and $5 for books and stationery. His account book shows the following entries:

CASH

1908	(*Receipts*)	*Dr.*		1908	(*Expenditures*)	*Cr.*	
Feb. 1	On Hand.....	$250	00	Feb. 1	By Insurance.....	$ 72	50
" 15	To Salary.....	200	00	" 28	" Household Exp.	110	00
" 27	" Rent.....	60	00		" Incidental "	25	50
					" Books and Stat.	5	00
					" Balance	297	00
		510	00			510	00
" 28	On Hand.....	297	00				

1. With what items is cash the man's debtor? With what items is cash his creditor?

2. During the month, how much money did the man receive? How much money did he actually possess on February 28 after paying his expenditures?

3. How much was spent? What is the difference between receipts and expenditures? How much more was spent than was received as salary?

Draw forms and prepare cash accounts for the following:

4. March, 1908. On hand first of the month $1000. March 5, sold land for $1500; received on the 15th, $150 for rent, and sold a team for $225 on the 25th. Bought real estate on the 8th for $1000; paid $65.25 for repairs on the 15th; taxes on the 16th amounted to $17.50; household expenses for the month amounted to $100, and personal expenses to $50.

5. June, 1908. A boy's receipts and expenditures were as follows: Received 25¢ for mowing the lawn on the 1st; 50¢ a day for running errands on the 4th and 5th; 25¢ for

mowing the lawn again on the 15th. On the 17th he paid $1.25 for a hat. The same day he earned 50¢ for delivering packages for the grocer.

6. July, 1908. A farmer sold on the 13th two loads of hay at $20 a load; 4 hogs weighing in all 1000 lb. at 6¢; 15 lb. butter at 20¢. On the 13th he bought 26 lb. granulated sugar at 6¢; 3 lb. coffee at 35¢; and dry goods to the amount of $15. On the 26th he sold vegetables to the amount of $1.25, and 9 doz. eggs at 10¢. The same day he bought clothing to the amount of $10.

7. Find cost of each purchase, also the amount due, on the bill below:

E. S. WICKWIRE,

1908 *In account with* MANNHEIMER BROS., *Dr.*

Jan.	4	To 1 Rug.....at	$75.50			
"	4	" 1 Overcoat..... "	35.00			
"	12	" 7 yd. Dress Goods "	2.00			
"	14	" 3½ yd. Silk..... "	1.50			
		Amount due.....				

Received payment,

Jan. 31, 1908. MANNHEIMER BROS.

Draw forms and prepare bills for the following accounts:

8. Edward Morris in account with J. W. Bowlby, Dr., St. Paul, Minn., May 30, 1908. 1 doz. collars at 12½¢ a piece; ½ doz. pr. cuffs at 25¢ a pair; 2 neckties at 50¢; 3 shirts at $1.25; 1 hat at $3.50; 1 pr. shoes at $3.50; 1 pr. gloves at $1.25. Paid, May 30.

9. A. H. Simons bought of Yerxa Bros., Chicago, June 6, 1908, 1 sack appleblossom flour at $1.30; 8 lb. ham at

17¢; 15 lb. granulated sugar at 6¢; ½ lb. Oolong tea at $1.60; 3 lb. Mocha and Java coffee at 40¢; 1 bunch of celery at 30¢. Paid in full the same day.

10. Edward Ryan bought of M. J. Doran, February 7, 1908, 2 cords of hard wood at $8.75; 3 T. of hard coal at $8.25; 2 cords of pine slabs at $3.25.

11. May 4, 1908, Alvin Johnson bought of the John Martin Lumber Co., cash payment, 1000 cedar shingles at $3 per M; 1 bbl. of lime at 68¢; 1000 ft. of pine flooring at $35 per M; 3000 laths at $1.50 per M; 15 lb. shingle nails at 7¢ per pound.

Accounts are not always settled in full at the close of the month. When a portion of the amount due is carried over to the next month, this item appears as the first item on the bill, under the title "account rendered."

12. Find amount of this bill:

J. FIRESTONE,

1908 *To* ALLEN, MOON & Co., *Dr.*

Feb. 1	To Account rendered.....	.....	.....	$24	85
" 10	" 2 Sacks of Flour @ ..	$2.60			
" 10	" 25 lb. Ham @	.18			
" 10	" 4 doz. Eggs @	.22			
" 10	" 6 lb. Butter @	.25			
" 15	" 25 lb. Granulated Sugar @	.06			
" 15	" Vegetables	5.00			
	By Cash.....				

13. A. L. Parker, in account with Marshall Field, lacks $12.75 of paying the entire bill of February, 1908. His March purchases were as follows: March 5, ½ doz. towels, @ 50¢ apiece; 2 table cloths, @ $6.50; 1 doz. napkins, @ $5.50 a doz.; 2 bed spreads @ $2.50; ½ doz. sheets @ 75¢

apiece; ½ doz. pillow cases @ 20¢ apiece; 7 yd. dress goods @ $1.50; findings, $5.25. Paid in full March 28.

14. Edward D. Young in account with F. W. Salisbury, wood and coal dealer, has left over from a previous account $8.25. On the 3d of January, 1908, he bought 2 T. of hard coal @ $8.25; 1 cord of hard maple wood, $8.50; 1 cord of pine slabs, $3.50. Paid the whole amount January 29.

15. On Nov. 15, Mrs. Jones purchased 1 can lima beans @ 12¢; 1 can peas @ 12¢; 1 qt. cranberries @ 12¢; 4 lb. pork chops @ 12¢; 4½ lb. chicken @ 12¢; 2 lb. butter @ 32¢, and paid the account with a $5.00 bill. What change was due her?

§50. Problems of the Grocery Clerk.

A grocery boy made the following transactions (sales and purchases), including the filling of the orders, and returned the correct change without an error all in one hour. What change did he return to each customer?

1. The first customer bought

Celery	$0.05	Potatoes	$0.15
Eggs	.22	Sugar, 10 lb. @	.04½

and paid the clerk with a $1.00 bill.

2. The second customer bought

Salt	$0.10	Apples	$0.25
Flour	.55	Breakfast food	.13
Yeast	.02	Rice, 3 lb. @	.08⅓
Milk	.07		

and he paid with $1.50.

3. The third customer bought

Tomatoes	$0.20	Beans, 4 qt. @	$0.07
Soap	.25	Tea, 2 lb. @	.60
Butter, 3 lb. @	.27	Cauliflower 3 heads @	.18
Coffee, 3 lb. @	.33⅓		

and paid with a $5.00 bill.

4. The fourth customer bought

Eggs	$0.22	Lampwicks	$0.10
Bacon	.18	Cheese	.16
Peas	.13		

and paid with a $5.00 bill.

5. The fifth customer bought

Cornmeal	$0.25	Melon	$0.25
Sweet potatoes	.27	Pineapple	.20
Candy	.10	Lard, 3 lb. @	.13⅓
Grapes	.18		

and paid with a $2 bill.

6. The sixth customer was a farmer, who sold to the grocer

Eggs, 6 doz. @	18¢	Tomatoes, 2 bu. @	75¢
Butter, 15 lb. @	22¢	Sugar corn, 6 doz. @	10¢
Potatoes, 5 bu. @	35¢		

and bought of the grocer

Sugar, 20 lb. @	4½¢	Tea, 2 lb. @	55¢
Coffee, 3 lb. @	25¢	Flour, 1 sack,	$1.15
Cheese, 2 lb. @	18¢	Rice, 6 lb. @	8⅓¢

How much money should the grocer pay the farmer to balance the account?

7. A grocer's daily sales for 4 weeks are given below. Without rewriting the numbers find the total sales (*a*) for each week; (*b*) for the 4 weeks; (*c*) the average daily sales for the month.

Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Weekly Totals
$28.75	$37.25	$35.18	$68.12	$20.13	$86.58	.....
18.62	9.68	21.83	40.28	37.60	75.76	.....
30.18	29.95	23.61	9.61	10.84	68.94	.....
19.27	39.13	28.16	38.19	5.63	98.56	.....

§51. Family Expense Account.

Foot the following problems as rapidly as you can work accurately.

1. The household expenditures (money spent) of a family are here given for each of the first 7 da. of Oct., 1900. Find the daily expenditures and the total expenditure for the week.

Oct. 1	
Oatmeal	$0.11
Meat	.18
Peaches	.25
Grapes	.13
Dried beef	.10
Tea	.30
Bread	.20
Celery	.05
Soap	.25
Total	

Oct 2	
Laundry	$0.48
Sweet potatoes	.18
Apples	.25
Bread	.10
Meat	.20
Sundries	.63
Bread	.20
Crackers	.15
Total	

Oct. 3	
Bread	$0.15
Baking powder	.25
Cheese	.10
Macaroni	.15
Bananas	.15
Honey	.20
Peanuts	.05
Tomatoes	.15
Shoe polish	.10
Car fare	.10
Onions	.10
Total	

Oct. 4	
Bread	$0.10
Meat	.10
Lunch boxes	.26
School supplies	.75
Car fare	.05
Milk	3.00
Apples	.20
Coffee	.35
Potatoes	.15
Beefsteak	.35
Tomatoes	.10
Servant	3.50
Candy	.05
Total	

Oct. 5	
Bread	$0.10
Meat	.10
Car fare	.35
Cakes	.10
School supplies	4.50
Car fare	.10
Meat	.23
Celery	.05
Tomatoes	.05
Cakes	.15
Sundries	.57
Car fare	.25
Beans	.10
Total	

Oct. 6	
Bread	$0.20
Bacon	.15
Fruit	.40
Merchandise	5.25
Meat	.42
Eggs	.20
Theatre	1.50
Bonbons	.10
Newspaper	.02
Sundries	.35
Total	

Oct. 7.	
Newspapers	.10

	Totals
Oct. 1	
Oct. 2.....	
Oct. 3.....	
Oct. 4.....	
Oct. 5.....	
Oct. 6.....	
Oct. 7.....	
Total for wk.....	

§52. The Equation.

1. y lb. of sugar are placed on the left scale pan, and a weight of 10 lb. on the right pan balances it (see Fig. 1). How heavy is y?

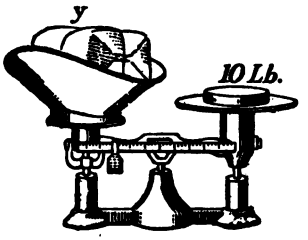


Figure 1

The balance of these two weights is expressed thus:

$$y = 10. \qquad \text{(I)}$$

This expression is called an equation, and is read "y equals 10."

2. If a 4-lb. weight is now added to the right pan, how many additional pounds of sugar must be placed upon the left pan to balance the scales? Write an equation to express the relation between the weights now in the pans.

3. If x lb. on the right balance y lb. on the left, how would you state the fact in an equation?

4. If 20 lb. are added to the x lb. already on the right, how many pounds must be added on the left to balance the equation?

5. If 35 is added on the right side of equation (I), what change must be made on the left side to balance the equation? Write the equation thus changed.

Just as the horizontal position of the scale beam shows that there is a balance of the weights on the pans, so the equality sign, $=$, shows that there is a *balance of value* of the numbers between which it stands. It must never be used between two numbers, or combinations of numbers, that are not equal in value.

The number on the left of the sign of equality is called the first member, or the left side of the equation. The number on the right is called the second member, or the right side of the equation.

6. Four equal weights and a 5-lb. weight just balance 21 pounds (Fig. 1, p. 186). How heavy is one of the equal weights?

SOLUTION: $4x + 5 = 21$. Subtract 5 from both sides.

$\underline{\quad 5 \quad 5}$

$4x \quad = 16$. If $4x = 16$, what is x? What is the answer to the problem?

Find the value of the letter

7. $3y + 2 = 14$ **9.** $7x + 8 = 29$ **11.** $15a + 3 = 48$

8. $5x + 12 = 32$ **10.** $9z + 3 = 75$ **12.** $12x - 9 = 27$

SOLUTION: $12x - 9 = 27$. Add 9 to both sides.

$\underline{\quad 9 \quad 9}$

$12x \quad = 36$. If $12x = 36$, one x, or x, equals what?

13. Solve for y:

(1) $22y - 6 = 38$ (5) $25y + 12 = 87$

(2) $18y - 9 = 45$ (6) $17 + 27y = 98$

(3) $28y - 6 = 50$ (7) $36y - 20 = 124$

(4) $30y - 10 = 50$ (8) $45y - 42 = 93$

14. Solve for t:

(1) $3t + 7 = 28$ (5) $32t - 4 = 60$

(2) $5t - 8 = 17$ (6) $48t + 6 = 102$

(3) $9t + 5 = 41$ (7) $42t - 5 = 79$

(4) $12t - 8 = 40$ (8) $39t + 7 = 124$

§53. Exercises for Practice.

1. Add until you can get correctly the sum for any one of these exercises in one minute:

(1)	(2)	(3)	(4)	(5)
3786	803	2863	30642	12
309	26099	18	96	864216
9167	1807	5	2875	7
43	391	680	758	3864
168	8786	85316	6	8
9	67	445	6412	4
28	732	78	7	7
887	6	8029	45369	629
21	42183	66	801	139621
4	27	3	94	5

2. Subtract, and prove by adding the difference to the subtrahend:

(1)	(2)	(3)	(4)	(5)
60821	164021	286002	780004	800103
5908	96479	190884	89746	73846

3. Two 3's are 6. How many 3's in 3 sixes? In 4 sixes? In 7 sixes? In 20 sixes?

4. Two 6's are 12. How many sixes in 4 twelves? In 6 twelves? In 10 twelves? In 18 twelves?

5. $2x = y$. How many x's make $4y$? $6y$? $5y$? $9y$? $15y$? $20y$? $30y$?

6. Three 8's are 24. How many 8's in 3 24's? In 5 24's? In 12 24's? In 20 24's?

7. Three 9's are 27? How many 9's in 4 27's? In 8 27's? In 7 27's? In 20 27's?

8. Multiply the following numbers mentally, writing only the products:

(1) 644×25	(4) 862×11	(7) $606 \times 66\frac{2}{3}$
(2) 748×50	(5) 386×250	(8) 328×125
(3) $832 \times 12\frac{1}{2}$	(6) $279 \times 33\frac{1}{3}$	(9) 245×98.

9. Solve the following problems mentally, writing only the quotients:

(1) $6400 \div 25$	(4) $35000 \div 500$	(7) $1640 \div 82$
(2) $6900 \div 300$	(5) $2600 \div 130$	(8) $1960 \div 98$
(3) $7500 \div 150$	(6) $8100 \div 90$	(9) $2180 \div 109$.

MEASUREMENT

§54. Measuring Value.

Money is the common measure of the value of all articles that are bought and sold.

The unit on which United States money is based is the dollar. This unit is called the U. S. standard of value.

1. A dollar is worth as much as how many dimes? Nickels? Quarters? Cents?

To measure the value of one amount of money by another is to find how many times one of the amounts is as large as the other.

2. Measure \$1 by 50¢; by 25¢; by 20¢; by 40¢; by 75¢.

3. Measure \$10.50 by 50¢; by 75¢; by \$1.50; by \$5.25.

4. A farm is worth \$1000 and a city lot \$2000. Measure the value of the farm by the value of the lot.

5. Measure the value of a \$150 horse by the value of a \$15 pig.

6. Measure \$75 by \$5; by \$25; by \$15; by \$150; by \$225.

7. Measure the value of 160 A. of land worth $75 per acre by $100; by $500; by $1000; by $8000; by $24,000.

8. Which is worth the more, 80 A. of land @ $75, or 50 A. @ $112? How much more?

9. Measure $400 by the value of an $8 calf; of a $25 colt.

§55. Measuring Length and Distance.

Answer these problems, first, by estimate, writing your estimates in a notebook; then answer by actual measurement. Finally, compare your estimates with the results of your measurements and tell how much your estimates differ from the measurements.

1. How wide is the page of this book? How long? How wide are the margins?

2. How wide is a pane of glass in your schoolroom window? How long is it?

3. How wide is the top of your desk? How long? How high is the top of your desk from the floor? How high is your seat?

4. How high is the bottom of the blackboard from the floor? How wide is your blackboard? How long is it?

5. How tall are you? How far can you reach by stretching both arms as far apart as possible?

6. How far is it around your waist? How far is it around your chest, just below your arms, when as much of the air as possible is exhaled from your lungs? What is your chest measurement when as much air as possible is drawn into your lungs?

7. The difference between these two chest measures is your chest expansion. How much is your chest expan-

sion? How does your chest expansion compare with the average for the pupils of your room?

8. How many steps wide is your schoolroom? How many steps long?

9. How many feet long is your room? How wide?

10. How many yards long is it? How many yards wide?

11. How many inches long is your step? How many feet long? How many yards long?

12. How many inches long and wide is your schoolroom?

13. How many feet long is your schoolhouse? How many yards long?

14. How many steps wide and how many steps long is your school yard? How many feet wide and how many feet long? How many yards wide and long?

15. How many steps is it from your home to the schoolhouse? How many feet? How many yards? Rods?

16. How far can you walk in 1 min.? How many miles could you walk in 1 hr. at the same rate?

17. How far is it from your home to a neighboring large city? Answer this by using a map and the scale given with it. How long would it take you to walk to that city at the rate of walking in problem 16?

18. How long will it take a train to run from your nearest station to that city at 24 mi. per hour?

19. Using the scale map of your state (see Geography) find the length and the width of your state; of your county; of the U. S.

20. How long and how wide is a two-cent postage stamp?

21. How long is the *diameter* (distance across) of a copper cent? Of a dime? Of a nickel? Of a quarter-dollar? Of a half-dollar? Of a silver dollar?

22. How long is the *radius* (half the diameter) of each of these coins?

23. Wrap a strip of paper, or a string, around each of these coins and find how far it is around each.

24. What kind of unit do you need to measure and express long distances? Medium distances? Short distances?

NOTE.—Such a number as three-eighths, or $\frac{3}{8}$, of an inch means three units, each one-eighth of an inch long. Such a unit is called a *fractional unit*, and such numbers as $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $1\frac{1}{8}$, $1\frac{7}{8}$, $2\frac{5}{7}$, are all said to be numbers expressed in fractional units. Name the unit of each of the six fractions just given.

25. How long is the distance around (*circumference*) a bicycle wheel? How long is the diameter of the wheel?

26. Answer the same questions for a carriage wheel; for the bottom of a bottle; of a can; for the head of a barrel; for any other circles you can find.

27. Arrange the measures found in problems 21–26 in a table like the one here given and keep them for future use. Thus:

OBJECT	DIAMETER	CIRCUMFERENCE
Silver dollar		
Half-dollar		
Quarter-dollar		
Nickel		
Dime		
Cent		
Bicycle wheel		
Carriage wheel		
Bottle		
Can		
Barrel head		

§56. Measuring Surfaces.

A square unit is a square 1 unit long and 1 unit wide.

1. The side wall of a room is 9 ft. by 12 ft. What will it cost to lath and plaster the wall at 33¢ per square yard?

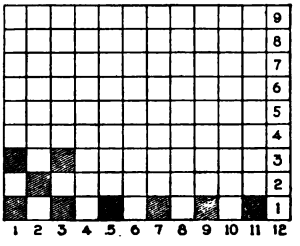


FIGURE 1

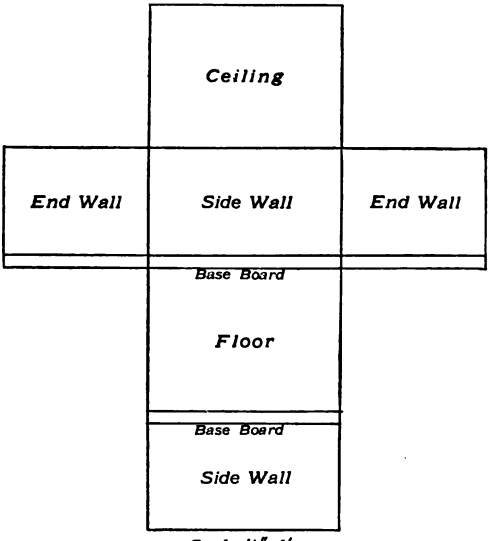
2. Using your own measures of the side and the end walls of your schoolroom, answer the same question for its walls and ceiling.

3. How many square feet of blackboard surface are there in your room?

4. How many square inches are there in a pane of glass in one of your windows? How many square feet of window surface admit light into your room?

5. This should be at least as great as $\frac{1}{6}$ of the floor surface of your room. Is it?

6. $\frac{1}{16}$ of an inch in the drawing, Fig. 2, stands for one foot in the room. Measure and find the number of



Scale 1/16=1'
FIGURE 2

square feet of plastered surface in the whole interior of the room, deducting the part covered by the baseboard and the floor.

Such a representation of the room as that of Fig. 2, p. 193, showing the walls and the ceiling spread out on a flat surface, is called a development of the room.

7. Draw to scale, from your own measures, the development of your schoolroom.

8. A schoolroom is 24′ square and 15′ high. How many sq. yd. of flooring are needed for the room? How many sq. yd. of plastering are needed for the ceiling?

9. One side wall (problem 8) contains two windows, each 6′ × 8′; the other has a window and a door, each 4′ × 8′; an end wall contains a window $6\frac{1}{2}$′ × 8′. How many sq. yd. of window and door surface in the entire room?

10. The rest of the wall surface is plastered. How many square feet of plastering are there in the room?

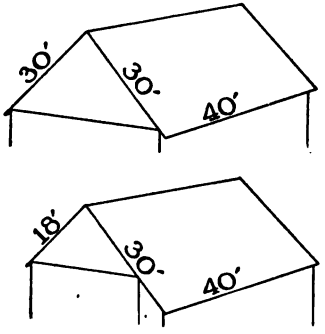


FIGURE 1

11. A barn roof is made up of two rectangles, each 30′ × 40′. How many square feet are there in the roof? How many squares of roofing? (A square of roofing is a 10-ft. square = 100 sq. ft.).

12. A house roof is made up of two rectangles, one 18′ × 40′ and the other 30′ × 40′. How many squares of roofing in the whole roof? How many sq. yd. of roof?

13. How many acres (1 acre = 160 sq. rd.) are there in a field 40 rd. × 80 rd.? In one 60 rd. × 80 rd.? 80 rd. × 80 rd.? 75 rd. × 120 rd.?

14. What is the cost at \$2.20 a sq. yd. of paving a street 60′ wide and $\frac{5}{8}$ of a mile long?

15. What is the cost at 15¢ per sq. ft. of making a cement sidewalk $1\frac{2}{3}$ yd. wide and $\frac{3}{8}$ of a mile long? $1\frac{3}{4}$ yd. wide and $\frac{5}{8}$ of a mile long?

16. The floor of an L-shaped basement is made of two rectangles, one of which is 9 yd. × 5 yd., and the other 5 yd. × 3 yd. How many square yards does it contain? How many square feet? What would it cost to cement it at 14¢ a square foot?

17. How many square feet of glass are there in a window containing 4 panes of glass, each 18″ × 1 yard?

18. A rug 9′ × 12′ is placed in a room 14′ × 18′, with its long edge parallel to the long side of the room and with its center at the middle of the floor. How many square feet of the floor remain uncovered?

19. The streets of a city cut out a row of blocks like that of Fig. 1. The numbers along the sides are the lengths in feet. How many square feet are there in block *A*? In *B*? In *C*?

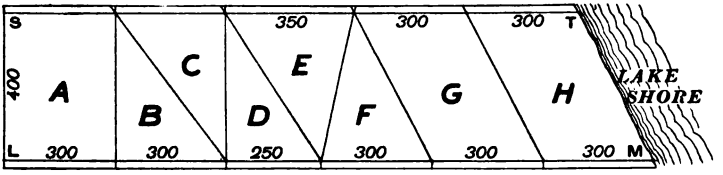


FIGURE 1

A four-sided figure, like A, having square corners, is called a rectangle.

A four-sided figure, like G, or H, having two pairs of parallel sides, whether the corners are square, or not, is a parallelogram.

20. What is the area of the rectangle of Fig. 1?

21. Examine the parallelograms and the rectangles beneath them and find the areas of the parallelograms.

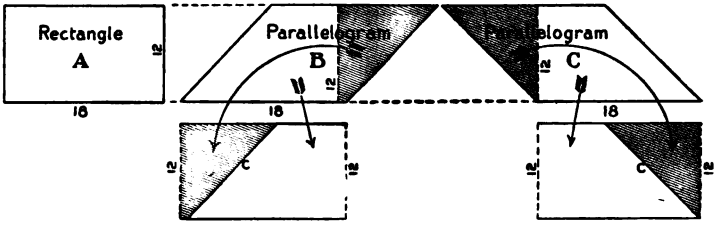


FIGURE 1

22. If the length of any parallelogram and its distance square across (*altitude*) are given, how can we find a rectangle with the same area as the parallelogram?

23. Point out in Fig. 1, p. 195, a rectangle that has the same area as the parallelogram *G*.

24. The length of a parallelogram is *b* ft. and the altitude is *a* feet. What is the area?

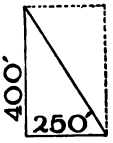


FIGURE 2

25. The streets, *ST* and *LM*, are parallel, Fig. 1, p. 195. How many square feet are there in block *G*? In *H*? In *E*? In *F*?

26. Study Fig. 2 and find how many square feet there are in *D* of Fig. 1, p. 195.

27. How can you find the area of a triangle when you know the length of its base and its altitude (shortest distance to the base from the opposite corner)?

28. In the triangle, *ABC* (Fig. 3), call the area *T*, the altitude (*CE*) *a*, the base (*AB*), *b*. Write an equation to find *T* from *a* and *b*.

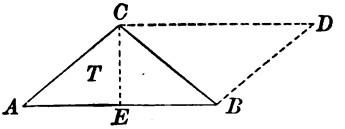


FIGURE 3

29. Find the area in square feet of the following triangles of Fig. 1, p. 195: *B*; *C*; *D*; *E*; *F*.

30. The part of a roof in the shape of a parallelogram has a length (*base*) of 18 ft. and a breadth (*altitude*) of 12 ft. What is the area of the parallelogram?

31. A city lot has the shape of a parallelogram of 400 ft. base and 300 ft. altitude. What is the area?

32. A field has the form of a triangle of 68 rd. base and 46 rd. altitude. What is the area?

33. Find the missing part (base, or altitude, or area) of the following parallelograms:

	Base	Altitude	Area
(1)	28 ft.	18 ft.	_____
(2)	40 rd.	_____	960 sq. rd.
(3)	_____	40 rd.	1600 sq. rd.
(4)	160 rd.	$60\frac{3}{4}$ rd.	_____
(5)	_____	88 rd.	28160 sq. rd.
(6)	*m* rd.	*n* rd.	_____

§57. Measuring Volume (Bulk) and Capacity.

Answer all you can orally.

1. How many cubic inches are there in a 2-inch cube? A 4-in. cube? An 8-in. cube? A 16-in. cube?

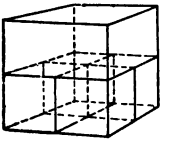


FIGURE 1

2. How many cubic feet are there in a cubic yard? In a 6-ft. cube? (See Fig. 2). In a room 3 yd. × 3 yd. × 3 yd.?

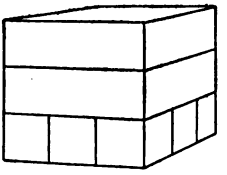


FIGURE 2

3. How many cubic inches are there in a 5-in. cube? In a 10-in. cube?

4. How many cubic inches are there in a 9-in. cube? In an 18-in. cube?

Use pencil and paper only when needed.

5. How many cubic inches are there in a cubic foot?

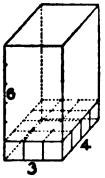


FIGURE 1

6. What is the capacity of a square-cornered box $3'' \times 4'' \times 6''$ (see Fig. 1)? $3' \times 4' \times 6'$? Of a room 3 yd. $\times$ 4 yd. $\times$ 5 yd.? a yd. $\times$ b yd. $\times$ c yards?

7. A box-car $8' \times 34'$ can be filled with wheat to a height of 5 ft. When full how many cubic feet of grain does it hold?

8. If a bushel of wheat $= \frac{5}{4}$ cu. ft., how many bushels does the car hold?

9. A cubic foot of grain (not ear-corn) is $\frac{4}{5}$ of a bushel. Make a rule for finding the number of bushels a wagon-box or a granary will hold when full.

10. A wagon-box is $2' \times 4' \times 10'$. How many bushels of ear-corn will it hold if $\frac{9}{4}$ cu. ft. $= 1$ bu. of ear-corn?

11. How many rectangular solids $4'' \times 3'' \times 7''$, will fill a box $16'' \times 12'' \times 28''$? (See Fig. 2).

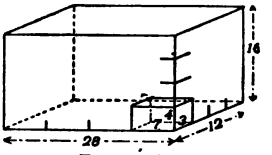


FIGURE 2

12. A box $30'' \times 24'' \times 12''$ will contain how many blocks $6'' \times 4'' \times 3''$ inches?

13. 128 cu. ft. $= 1$ cord. How many cords in a straight pile of wood 80 ft. $\times$ 4 ft. $\times$ 4 feet?

14. The volume of a rectangular bin is 1500 cu. ft. If it is 25 ft. long and 6 ft. deep, what is its width?

15. How many bricks $8'' \times 4'' \times 2''$ are there in a regular pile $218'' \times 24'' \times 48$ inches?

16. Measure the length, the width, and the height of

your schoolroom, and find how many cubic feet of air it contains.

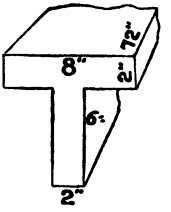


FIGURE 1

17. How many pounds does the steel T-beam of Fig. 1 weigh if 1 cu. in. of steel weighs $4\frac{1}{2}$ ounces?

18. A steel I-beam 24′ long has a cross section of the form and size shown in Fig. 2. How much does it weigh if steel weighs 486 lb. per cubic foot (metal 1″ thick)?

FIGURE 2

19. A water tank is $3' \times 6' \times 10'$. How many cubic feet of water does it hold when full?

20. There are 231 cu. in. in a gallon; how many gallons does the tank of problem 19 hold?

21. How many cubic inches of grain does a box hold if it is 8 in. square and $4\frac{1}{8}$ in. deep? How many gallons?

22. How many cubic inches does a box hold if it is 16 in. square and $8\frac{1}{2}$ in. deep? How many bushels (2150.42 cu. in. = 1 bushel)?

23. How many cubic inches does a box hold if it is $9'' \times 10'' \times 12''$? About what part of a bushel?

24. Answer similar questions for a box 10 in. square and $10\frac{3}{4}$ in. deep; for a box $10'' \times 12'' \times 18''$.

§58. Measuring Weight.

1. A 3-in. cube of soil weighs 20 ounces; how many pounds does 1 cu. ft. of the same soil weigh? How many pounds does a cubic yard weigh?

2. A 4-in. cube of sand weighs 48 ounces; how many pounds does 1 cu. ft. of the same sand weigh? How many pounds does a cubic yard weigh?

3. A cubic foot of water weighs 1000 ounces; how much does a 6-in. cube of water weigh? A 3-in. cube?

4. A cubic foot of iron weighs 480 pounds; how many ounces does 1 cu. in. weigh?

SUGGESTION: Notice that 1 cu. ft. = 12 × 12 × 12 cu. in., then use cancellation, thus:

$$\frac{\overset{10}{\cancel{480}}\times\overset{4}{\cancel{16}}}{\underset{3}{\cancel{12}}\times\underset{3}{\cancel{12}}\times\cancel{12}}=\frac{10\times4}{3\times3}=\frac{40}{9}=4\tfrac{4}{9}.$$ Ans. 1 cu. in. weighs $4\frac{4}{9}$ oz.

5. A cubic foot of oak wood weighs 64 lb.; how many ounces does 1 cu. in. of oak wood weigh? How many cubic inches of oak weigh just 1 pound?

6. A cubic foot of copper weighs 552 lb.; how many ounces does 1 cu. in. of copper weigh? How many cubic inches of copper weigh a pound?

7. A cubic foot of gold weighs 1200 lb.; how many ounces does 1 cu. in. of gold weigh? How many cubic inches of gold weigh a pound?

8. A cubic foot of zinc weighs 436 lb.; how many ounces does 1 cu. in. of zinc weigh? How many cubic inches of zinc weigh a pound?

9. A bushel of coal weighs 80 lb., a short ton weighs 2000 lb., and a long ton weighs 2240 pounds. How many bushels of coal are there in a car load of 36 short tons? Of 36 long tons?

10. How many tons of timothy hay will fill a hay-mow, $18' \times 23' \times 40'$, counting 460 cu. ft. to a ton?

11. How many 100-lb. bales of hay, each $1\frac{1}{2}' \times 2' \times 3\frac{1}{2}'$, will it take to fill a car $8' \times 9' \times 42'$? Find the weight.

§59. Measuring Temperature.

On the Fahrenheit thermometer the point where water begins to freeze is marked 32°, and the point where water begins to boil is marked 212°.

1. How many degrees are there between the boiling point and the freezing point?

2. At 11 P.M. on a certain date a thermometer read 32°. The mercury then fell $\frac{1}{2}$° an hour for 4 hours. What was the reading at 3 A.M. the next day?

3. The mercury then fell 1° an hour for 5 hours. What was the reading at 8 A.M.?

4. At 3 A.M. on a certain day the reading was 26°. The mercury fell 2° an hour for 5 hours. What was the reading at 8 A.M.?

5. On a certain date the reading was 10°, and the mercury fell on the average $1\frac{3}{7}$° an hour for 7 hours. What was the reading at the end of the 7 hours?

6. It then fell $1\frac{1}{4}$° an hour for 8 hours. What was the reading at the end of the 8 hours?

7. The mercury fell from the reading 12° above zero to 3° below zero. How many degrees did it fall?

8. The mercury rose regularly from 10° below zero to 5° above zero in 3 hours. How many degrees per hour did it rise? At the same rate what would the reading be 2 hours later?

9. We might write readings *above zero* thus: A. 12°; A. 6°; A. 32°; and readings *below zero* thus: B. 3°; B. 6°; B. 30°. How many degrees does the mercury fall in the following pairs of readings:

(1) A. 8° to A. 3°
(2) A. 8° to B. 3°
(3) A. 2° to B. 12°
(4) B. 2° to B. 11°

10. How many degrees of rise or fall are there in the following pairs of readings? If the change is a *rise*, mark it R; if a *fall*, mark it F:

(1) B. 7° to B. 2°
(2) B. $2\frac{1}{2}°$ to B. 1°
(3) B. $2\frac{1}{2}°$ to A. 1°
(4) A. 9° to B. 9°
(5) A. $2\frac{1}{2}°$ to B. $2\frac{1}{2}°$
(6) A. $6\frac{1}{2}°$ to A. 12°
(7) A. 3° to B. $30\frac{1}{2}°$
(8) A. 18° to A. $67\frac{1}{2}°$

11. Instead of writing an A. for 'above zero' readings, it is customary to use the sign (+). What sign would you then suggest for 'below zero' readings? Tell whether a change from the first to the second in the following pairs of readings denotes a *rise* or a *fall* in each case and by how much.

(1) + 16° to + 100°
(2) + 32° to + 212°
(3) + 32° to − 3°
(4) + 16° to − 17°
(5) − 18° to − 34°
(6) − 18° to − $6\frac{1}{2}°$

12. The 12 o'clock (noon) readings for 4 successive days were as follows: $+ 82\frac{1}{2}°$ $+ 78\frac{3}{4}°$ $+ 61\frac{1}{2}°$ $+ 53\frac{1}{4}°$ What is the average of these 12 o'clock readings?

13. Find the average of these 9 A.M. readings for 6 da.: $+ 9°$ $+ 4°$ $+ 5°$ $+ 12\frac{1}{2}°$ $+ 14\frac{1}{4}°$ $+ 9\frac{1}{4}°$

14. Find the average of these 6 readings: $- 4°$ $- 6°$ $- 2°$ $- 5°$ $- 13°$ $- 12°$

15. Find the average of these 2 readings: + 8° and − 2°. Show on Fig. 1, p. 176, what point on the thermometer is midway between the readings + 8° and − 2°.

16. What point is midway between the readings + 13° and − 5°? Between the readings + 4° and − 6°? + 12° and − 12°? − 4° and − 12°?

§60. Measuring Time.

The primary unit used in measuring time is the *mean solar day*. This day is the average time interval during which the rotation of the earth carries the meridian of a place eastward from the sun back around to the sun again. It is the average length of the interval from noon to the next noon.

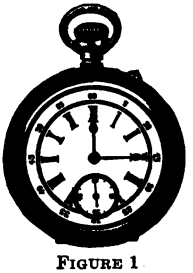


FIGURE 1

1. If the timepiece is running correctly, how many times does the short hand (the hour hand) of a watch or clock turn completely around from noon to the next noon?

2. How much time is measured by one complete turn of the hour hand?

3. What part of a day of 24 hr. is measured by the rotation (turning) of the hour hand from XII to VI? From XII to III? From XII to I?

4. What name is given to the time-interval in which the hour hand moves from XII to I?

5. For measuring shorter periods the motion of the long hand (the minute hand) is used. What part of a day is measured by the movement of the minute hand from XII around to XII again? From XII to VI? From XII to III? From XII to IX?

6. What name is given to the interval of time required for the long hand to make one complete turn?

7. How many times does the minute hand turn around while the hour hand turns around once?

8. The small and rapidly moving hand covering the VI of the watch face is the second hand. How much time is measured by one whole turn of the second hand?

9. Over how much space does the tip of the minute hand move while the second hand moves around once?

10. The second hand circle is divided into how many equal parts? What name is given to the time interval in which the second hand moves over one of these equal spaces?

11. How many times does the minute hand turn around in one day? In a week? In a month of 30 da.? In 365 da. (1 common year)?

12. Answer the same questions for the second hand.

13. There are about $365\frac{1}{4}$ da. in 1 yr. If the hour and the minute hands of a watch start at XII and the second hand at 60 at the beginning of a year, how will the hands point at the instant the year ends, if the watch runs correctly and without stopping?

COMMON FRACTIONS

§61. Reducing Fractions to Higher and Lower Terms.

1. Show from figures *B*, *G*, *C*, *D*, and *E* that

$$\tfrac{1}{2} = \tfrac{2}{4} \quad \tfrac{1}{2} = \tfrac{3}{6} \quad \tfrac{1}{2} = \tfrac{4}{8} \quad \tfrac{1}{2} = \tfrac{6}{12} \quad \tfrac{1}{2} = \tfrac{12}{24}$$

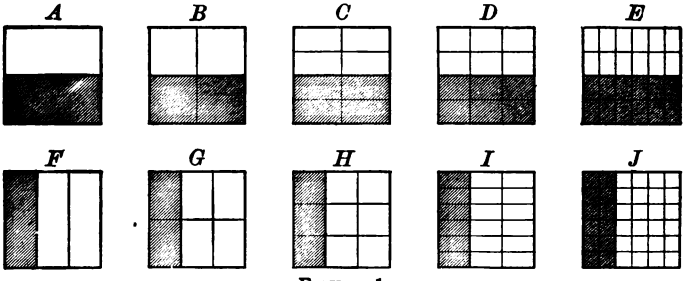


FIGURE 1

2. How could halves be expressed as 4ths without a picture? As 6ths? As 8ths? As 12ths?

3. Show from figures *G*, *H*, *D*, *I*, and *J* that

$\frac{1}{3} = \frac{2}{6}$ $\frac{1}{3} = \frac{3}{9}$ $\frac{1}{3} = \frac{4}{12}$ $\frac{1}{3} = \frac{6}{18}$ $\frac{1}{3} = \frac{12}{36}$

$\frac{2}{3} = \frac{4}{6}$ $\frac{2}{3} = \frac{6}{9}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{2}{3} = \frac{12}{18}$ $\frac{2}{3} = \frac{24}{36}$

4. How could 3ds be expressed as 6ths without a picture? As 9ths? As 12ths? As 18ths?

5. Draw figures and show that

$\frac{1}{4} = \frac{2}{8}$ $\frac{1}{4} = \frac{3}{12}$ $\frac{1}{4} = \frac{4}{16}$ $\frac{1}{4} = \frac{5}{20}$ $\frac{1}{4} = \frac{6}{24}$

$\frac{3}{4} = \frac{6}{8}$ $\frac{3}{4} = \frac{9}{12}$ $\frac{3}{4} = \frac{12}{16}$ $\frac{3}{4} = \frac{15}{20}$ $\frac{3}{4} = \frac{18}{24}$

6. How could fourths be expressed as 8ths without using a picture? As 12ths? As 16ths?

7. Tell what numbers should stand in place of the letter x, in the following:

$\frac{1}{5} = \frac{x}{10}$ $\frac{1}{5} = \frac{x}{15}$ $\frac{1}{5} = \frac{x}{20}$ $\frac{1}{5} = \frac{x}{25}$ $\frac{1}{5} = \frac{x}{30}$

$\frac{2}{5} = \frac{x}{10}$ $\frac{2}{5} = \frac{x}{15}$ $\frac{2}{5} = \frac{x}{20}$ $\frac{2}{5} = \frac{x}{25}$ $\frac{2}{5} = \frac{x}{30}$

$\frac{3}{5} = \frac{6}{x}$ $\frac{3}{5} = \frac{9}{x}$ $\frac{5}{5} = \frac{12}{x}$ $\frac{3}{5} = \frac{15}{x}$ $\frac{3}{5} = \frac{18}{x}$

$\frac{4}{5} = \frac{8}{x}$ $\frac{4}{5} = \frac{12}{x}$ $\frac{4}{5} = \frac{20}{x}$ $\frac{4}{5} = \frac{24}{x}$ $\frac{4}{5} = \frac{28}{x}$

8. Express as sixths: $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$

9. Express as twelfths: $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{1}{6}$ $\frac{5}{6}$

10. Express as eighteenths: $\frac{1}{9}$ $\frac{1}{6}$ $\frac{2}{9}$ $\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{5}{9}$ $\frac{7}{9}$

11. Express as halves: $\frac{2}{4}$ $\frac{3}{6}$ $\frac{5}{10}$ $\frac{6}{12}$ $\frac{7}{14}$ $\frac{8}{16}$ $\frac{10}{20}$

12. Express as thirds: $\frac{2}{6}$ $\frac{4}{6}$ $\frac{3}{9}$ $\frac{6}{9}$ $\frac{4}{12}$ $\frac{8}{12}$ $\frac{12}{12}$ $\frac{6}{15}$ $\frac{10}{15}$ $\frac{15}{15}$

13. How may 6ths be expressed as 3ds? 9ths as 3ds? 12ths as 3ds? 15ths as 3ds?

14. Express as 4ths: $\frac{2}{8}$ $\frac{4}{8}$ $\frac{6}{8}$ $\frac{3}{12}$ $\frac{6}{12}$ $\frac{9}{12}$ $\frac{12}{12}$ $\frac{4}{16}$ $\frac{12}{16}$ $\frac{5}{20}$ $\frac{15}{20}$

15. How may 8ths be expressed as 4ths? 12ths as 4ths? 16ths as 4ths? 20ths as 4ths?

The number above the line is the numerator (meaning numberer).
The number below the line is the denominator (meaning namer).

The numerator and denominator, when spoken of together, are called the terms of the fraction.

It has been seen that multiplying both terms of the fractions by the same number does not change the value of the fraction.

Problems 11 to 15 show that *dividing* both terms *by the same number* reduces to lower terms but does not change the value of the fraction.

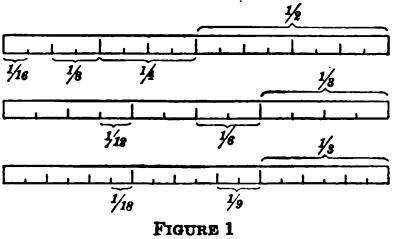


FIGURE 1

16. Show, from Fig. 1, how to express $\frac{1}{2}$ as 4ths; as 8ths; 16ths.

17. Express $\frac{1}{3}$ as 6ths; 9ths; 12ths; 18ths.

18. Express as 3ds: $\frac{4}{6}$ $\frac{6}{9}$ $\frac{14}{21}$ $\frac{20}{30}$ $\frac{140}{210}$ $\frac{26}{39}$

19. Express as 7ths: $\frac{4}{28}$ $\frac{35}{49}$ $\frac{24}{56}$ $\frac{54}{63}$

20. Compare the values of the following fractions after reducing each to its lowest terms:

$$\frac{3}{4} \quad \frac{6}{8} \quad \frac{9}{12} \quad \frac{21}{28} \quad \frac{60}{80} \quad \frac{75}{100} \quad \frac{300}{400}$$

21. Express $\frac{2}{5}$ as 10ths; 15ths; 20ths; 35ths; 75ths; 100ths.

22. By what must the numerator of $\frac{3}{4}$ be multiplied to make the numerator the same as that of $\frac{6}{8}$? By what must the denominator of $\frac{3}{4}$ be multiplied to make the denominator the same as that of $\frac{6}{8}$? Draw a square and divide it free-hand to show the comparative sizes of $\frac{3}{4}$ and $\frac{6}{8}$.

23. By what must each term of $\frac{3}{4}$ be multiplied to obtain $\frac{9}{12}$? $\frac{21}{28}$? $\frac{60}{80}$? $\frac{75}{100}$? $\frac{300}{400}$? What, then, is the *value* of each fraction of problem 20?

24. What is the value of all the fractions in each of the following sets?

(1) $\frac{2}{3}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{14}{21}$, $\frac{20}{30}$, $\frac{24}{36}$;

(2) $\frac{4}{5}$, $\frac{8}{10}$, $\frac{16}{20}$, $\frac{20}{25}$, $\frac{40}{50}$, $\frac{44}{55}$.

25. How may the value of a fraction be expressed in higher terms?

A fraction may be expressed in higher terms, without changing its value, by multiplying both terms by the same integer (larger than 1).

26. Express the following fractions in higher terms:

$$\frac{3}{8} \quad \frac{6}{7} \quad \frac{5}{9} \quad \frac{13}{15} \quad \frac{12}{24} \quad \frac{6}{12} \quad \frac{20}{50}$$

27. How can the terms of $\frac{3}{4}$ be obtained from the terms of $\frac{6}{8}$? Of $\frac{9}{12}$? Of $\frac{21}{28}$? Of $\frac{75}{100}$?

28. What fraction is obtained by dividing each term of $\frac{4}{28}$ by 4? Compare the value of $\frac{4}{28}$ with the value of the fraction $\frac{1}{7}$.

29. By using a rectangle, divided as in Fig. 1, show that $\frac{4}{28}$ of the rectangle equals $\frac{1}{7}$ of it.

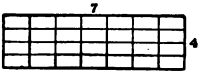


FIGURE 1

30. In what lower terms can $\frac{20}{30}$ be expressed? $\frac{40}{100}$? $\frac{30}{42}$? Show that $\frac{20}{30} = \frac{2}{3}$ by means of a properly divided rectangle.

31. What effect on the *value* of a fraction does dividing each of its terms by the same number have?

32. How may the value of a fraction be expressed in lower terms?

Multiplying or dividing both terms of a fraction by the same number changes the form of the fraction, but does not change its value.

Fractions are most easily used when their numerators and their denominators are the smallest possible whole numbers.

Fractions are in their lowest terms when their terms are the smallest possible whole numbers.

Among the many ways in which $\frac{12}{24}$ can be written are these:

$$\frac{12}{24} \quad \frac{6}{12} \quad \frac{3}{6} \quad \frac{2}{4} \quad \frac{1}{2}$$

33. Show how $\frac{1}{2}$ is obtained from $\frac{12}{24}$, using numbers only.

§62. Problems.

In each case when the answer to the problem is a fraction, or ratio, express it in its *lowest* terms.

1. A man works 48 da. out of 64 days. What part of 64 da. does he work?

2. A grocer bought 24 boxes of oranges and sold 16 boxes. What part of his purchase did he sell?

3. Out of 56 bu. of potatoes a huckster sold 49 bu. What part of his potatoes did he sell?

4. A street car on one line makes a weekly average of 72 trips. A car on another line makes an average of 144 trips. What is the ratio of the former to the latter?

5. A man pays \$50 for wood and \$125 for coal in one season. Find the ratio of the cost of the wood to the cost of the coal.

6. Out of 1000 ft. of lumber purchased, 500 ft. were used for the flooring of two rooms. Let x equal the ratio of the number of feet of lumber used to the total number of feet purchased. Find the value of x.

7. Out of 126 bu. of oats, a livery man fed 63 bu. in a week. Let y equal the ratio of the quantity of oats bought to the quantity of oats fed. Find y.

8. In 644 lb. of sandy loam there were 364 lb. of sand and gravel. What part of the soil, by weight, was sand and gravel?

9. The human body needs about 94.5 oz. of water and solid food each day, of which 64.8 oz. should be water. The water is what part, by weight, of the total quantity of solid food and water?

SUGGESTION: Both terms of the fraction may be multiplied by 10 without changing the value of the fraction. Then divide both new terms by 27.

10. The total population of a certain city is 36,729, of which 12,243 are colored and 10,017 are foreign. What part of the entire population is colored? What part is foreign?

SUGGESTION: Divide both terms of $\frac{12,243}{36,729}$ by 12,243 and both terms of $\frac{10,017}{36,729}$ by 3339.

One way of answering the question: "What part of the total population is made up of colored and of foreign persons?" is to find the sum $\frac{1}{3} + \frac{3}{11}$.

11. A boy spent $\frac{2}{5}$ of his money and gave away $\frac{1}{3}$ of it; what part of it had he left?

To solve such problems as 11 it is necessary to know how to add, and to subtract fractions.

ADDITION AND SUBTRACTION OF FRACTIONS

§63. Fractions Having a Common Fractional Unit (a Common Denominator).

The fraction $\frac{3}{4}$ means 3 of the $\frac{1}{4}$'s (one-fourths) of a number, or measured quantity. $\frac{1}{4}$ is called the fractional unit of $\frac{3}{4}$.

1. What are the fractional units of

$$\frac{4}{7}?\quad \frac{5}{9}?\quad \frac{2}{3}?\quad \frac{11}{12}?\quad \frac{8}{15}?\quad \frac{19}{26}?\quad \frac{32}{75}?$$

2. What is the unit of a fraction having 6 for its denominator? 7? 18? 25? 30? 50? 100?

3. Complete these equations:

$\frac{4}{9}$ yd. + $\frac{3}{9}$ yd. =	$\frac{3}{6}$ gal. + $\frac{2}{6}$ gal. =
$\$\frac{5}{10}$ + $\$\frac{8}{10}$ =	$\frac{5}{17}$ + $\frac{6}{17}$ =
$\frac{3}{7}$ wk. + $\frac{2}{7}$ wk. =	$\frac{7}{12}$ + $\frac{1}{12}$ =
$\frac{3}{5}$ bu. + $\frac{1}{5}$ bu. =	$\frac{5}{a}$ + $\frac{6}{a}$ =

NOTE: $\frac{5}{a}$ is read "5 divided by a."

4. Make a rule for adding (finding the sum of) two fractions having a common denominator.

5. Complete these equations:

$\frac{4}{9} - \frac{3}{9} =$	$\frac{8}{9} - \frac{5}{9} =$	$\frac{7}{15} - \frac{3}{15} =$	$\frac{11}{24}$ hr. $- \frac{5}{24}$ hr. =
$\frac{7}{10} - \frac{4}{10} =$	$\frac{5}{7} - \frac{2}{7} =$	$\frac{10}{12} - \frac{3}{12} =$	$\frac{13}{14}$ da. $- \frac{5}{14}$ da. =

6. Make a rule for finding the difference between two fractions that have a common denominator, that is, whose denominators are the same number.

The sum, or the difference, of any two fractions having common denominators equals the sum, or the difference, of their numerators, divided by the common denominator.

§64. Fractions Easily Reduced to Common Fractional Unit.

1. $\frac{1}{4}$ equals how many 12ths? 8ths? 16ths? 20ths? 64ths? 100ths?

2. $\frac{7}{12}$ hr. + $\frac{1}{4}$ hr. = x hr. What is the value of x?

SOLUTION: $\frac{7}{12}$ hr. + $\frac{3}{12}$ hr. = $\frac{10}{12}$ hr. = $\frac{5}{6}$ hr. $x = \frac{5}{6}$.

3. $\frac{7}{12}$ hr. $- \frac{1}{4}$ hr. = y hr. What is y?

4. Find these sums and differences:

$\frac{2}{5} + \frac{3}{10} = ?$	$\frac{6}{7} - \frac{3}{14} = ?$	$\frac{9}{11} + \frac{3}{22} + \frac{2}{33} = ?$
$\frac{1}{2} + \frac{3}{8} = ?$	$\frac{3}{8} - \frac{3}{16} = ?$	$\frac{13}{32} + \frac{5}{16} + \frac{3}{8} = ?$
$\frac{7}{9} - \frac{1}{3} = ?$	$\frac{7}{15} - \frac{1}{5} = ?$	$\frac{7}{14} + \frac{3}{28} - \frac{5}{42} = ?$
$\frac{5}{12} + \frac{2}{3} = ?$	$\frac{7}{12} - \frac{1}{3} = ?$	$\frac{8}{30} + \frac{2}{15} - \frac{1}{5} = ?$
$\frac{11}{12} - \frac{2}{3} = ?$	$\frac{10}{21} - \frac{2}{7} = ?$	$\frac{13}{24} - \frac{1}{10} + \frac{5}{6} = ?$

§65. Definitions and Principles.

Thus far we have had to do with (1) whole numbers, or integers, and (2) fractional numbers, or fractions.

A proper fraction is a fraction whose numerator is less than its denominator: *as* $\frac{2}{3}$, $\frac{5}{6}$, $\frac{2}{7}$.

An improper fraction is a fraction whose numerator is equal to, or greater than, its denominator: *as* $\frac{3}{3}$, $\frac{5}{2}$, $\frac{7}{3}$, $\frac{8}{2}$.

A mixed number is a number that is composed of an integer and a fraction: *as* $2\frac{1}{2}$, $12\frac{3}{4}$, $7\frac{5}{6}$.

1. Name the proper and the improper fractions, the mixed and the whole numbers in the following:

$$\frac{3}{4}\ \ \frac{3}{8}\ \ \frac{1}{16}\ \ 5\ \ \frac{12}{5}\ \ \frac{8}{2}\ \ \frac{2}{8}\ \ 1\frac{7}{8}\ \ 3\frac{1}{15}\ \ 6.75\ \ \frac{3}{15}\ \ 2$$

2. Make and give an example of each class.

3. How many times is 1 contained in 3? In any number? We may write this in symbols, thus:

$$\frac{3}{1} = 3 \qquad \frac{18}{1} = 18 \qquad \frac{25}{1} = 25 \qquad \frac{a}{1} = a$$

Any whole number may be written in the fractional form by writing 1 *for its denominator.*

4. Write the following numbers in fractional form:

$$12 \qquad 24 \qquad 32 \qquad 68 \qquad a \qquad x$$

5. How many halves are there in 1? In 2? In 5? In $2\frac{1}{2}$? In $5\frac{1}{2}$?

6. How many 5ths are there in 1? In 2? In 4? In 6? In $6\frac{2}{5}$? In $7\frac{3}{5}$?

7. Change these integers into 6ths: 2 5 16 120

8. Change the following into 6ths:

$$1\frac{5}{6} \qquad 3\frac{1}{6} \qquad 8\frac{1}{6} \qquad 16\frac{5}{6} \qquad 15\frac{1}{6}$$

9. Change the following into 7ths:

$$5 \qquad 6\frac{2}{7} \qquad 10\frac{5}{7} \qquad 15\frac{6}{7} \qquad 20\frac{3}{7}$$

10. Change the following numbers into 12ths: 2 8 12 20 56 128

11. How may any whole number be changed into 12ths? Into 9ths? Into 24ths? Into a fraction having any given denominator?

Any integer may be expressed in the form of a fraction having a given denominator by multiplying the integer by the given denominator and writing the product over that denominator.

12. Express 6, 18, 17, 22, 39, 28 as 4ths; as 7ths; as 10ths; as 100ths.

13. Change into 6ths: 2 $4\frac{1}{6}$ $24\frac{5}{6}$ $37\frac{5}{6}$

14. Change the following numbers into improper fractions having 7 for a denominator:

$$3\tfrac{1}{7} \quad 24\tfrac{2}{7} \quad 106\tfrac{3}{7} \quad 234\tfrac{6}{7} \quad 648$$

15. How can you change any mixed number into an improper fraction whose denominator is the denominator of the given fraction?

A mixed number may be expressed as an improper fraction by multiplying the whole number by the denominator, adding the numerator to the product, and writing the sum over the given denominator.

16. Reduce to improper fractions:

$$2\tfrac{1}{5} \quad 6\tfrac{1}{3} \quad 12\tfrac{1}{2} \quad 18\tfrac{3}{4} \quad 25\tfrac{6}{7} \quad 328\tfrac{5}{9} \quad 609\tfrac{11}{13}$$

17. Express $\frac{12}{6}$ as a whole number. Change $\frac{13}{6}$ to a mixed number.

18. Change to whole, or mixed, numbers the following improper fractions:

$$\frac{6}{3} \quad \frac{4}{3} \quad \frac{7}{2} \quad \frac{12}{7} \quad \frac{14}{7} \quad \frac{16}{7} \quad \frac{24}{3} \quad \frac{18}{5} \quad \frac{25}{9} \quad \frac{36}{11} \quad \frac{36}{12} \quad \frac{45}{9} \quad \frac{20}{7}$$

19. How may any improper fraction be changed to a whole, or mixed, number?

An improper fraction may be changed to a whole, or mixed, number by making the indicated division.

20. Reduce to whole, or mixed, numbers the following:

$\frac{4}{2}$ $\frac{16}{7}$ $\frac{28}{14}$ $\frac{706}{7}$ $\frac{95}{7}$ $\frac{343}{100}$ $\frac{1682}{13}$

§66. Applications of Addition and Subtraction of Fractions.

1. From the river, R, to the bend, A, the road is $1\frac{1}{4}$ miles long. From the bend, A, to the bend, B, is $\frac{1}{2}$ mile, and from bend, B, to the cross-road, C, is $\frac{3}{4}$ mile. How far is it, by the road, from R to C?

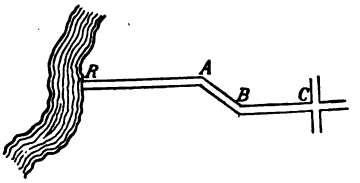


FIGURE 1

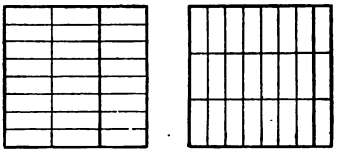
2. A man owns $\frac{2}{3}$ of an acre in one block and $\frac{7}{8}$ of an acre in another block. How much land does he own?

This problem requires us to add $\frac{2}{3}$ and $\frac{7}{8}$.

Fig. 2 (*a*) is divided by the vertical lines into 3 equal parts and by the horizontal lines into 8 equal parts. Into how many equal parts do the 2 sets of parallel lines divide the square (*a*)?

Show by Fig. 2 (*a*) that $\frac{2}{3}=\frac{16}{24}$.

Show by Fig. 2 (*b*) that $\frac{7}{8}=\frac{21}{24}$.



(*a*) (*b*)

$\frac{2}{3}=\frac{16}{24}$ $\frac{7}{8}=\frac{21}{24}$

FIGURE 2

$\frac{2}{3}+\frac{7}{8}=\frac{16}{24}+\frac{21}{24}=\frac{37}{24}=1\frac{13}{24}$. *Ans.* $1\frac{13}{24}$ acres.

What is the least common denominator of $\frac{2}{3}$ and $\frac{7}{8}$? What, then, is the largest fractional unit (the fractional unit having the least denominator) in which both $\frac{2}{3}$ and $\frac{7}{8}$ can be exactly expressed? $\frac{2}{3}=\frac{?}{24}$. $\frac{7}{8}=\frac{?}{24}$.

$\frac{2}{3}+\frac{7}{8}=\frac{16}{24}+\frac{21}{24}=\frac{16+21}{24}=\frac{37}{24}=1\frac{13}{24}$. *Ans.* $1\frac{13}{24}$ acres.

3. Add: $\frac{3}{4}$ and $\frac{2}{3}$ $\frac{4}{5}$ and $\frac{3}{7}$ $\frac{1}{4}$ and $\frac{3}{7}$ $\frac{7}{8}$ and $\frac{4}{9}$

4. In 1900, the New England states produced about $\frac{11}{24}$; the Middle Atlantic states about $\frac{9}{20}$, and the Central states about $\frac{1}{12}$ of all the maple sugar produced in the United States. About what part of the maple sugar did they all produce?

5. In 1900, the New England states produced about $\frac{1}{8}$ of the maple syrup of the country; the Middle Atlantic states about $\frac{2}{7}$, and the Central states about $\frac{4}{7}$ of it. About what part of the maple syrup of the country did the three groups produce?

6. A boy had $\$\frac{4}{5}$ and spent $\$\frac{1}{2}$. What part of a dollar did he have left?

The least common denominator is $2 \times 5 = 10$.

$\frac{4}{5}=\frac{8}{10}$; $\frac{1}{2}=\frac{5}{10}$. Then, $\frac{4}{5}-\frac{1}{2}=\frac{8}{10}-\frac{5}{10}=\frac{3}{10}$. *Ans.* $\frac{3}{10}$.

Show this solution also, using a divided rectangle.

7. William is $4\frac{2}{3}$ ft. tall and James is $4\frac{5}{12}$ ft. tall. Who is the taller and by how much?

8. A man owned $\frac{15}{16}$ A. of land and sold $\frac{3}{7}$ A. How much land did he then own?

9. From $\frac{5}{6}$ lb. of loam $\frac{2}{7}$ lb. sand was removed. How much of the loam remained?

10. A man having $\$4\frac{1}{5}$ paid a debt of $\$2\frac{3}{4}$. How much money had he after paying the debt?

SOLUTION: $\frac{1}{5}=\frac{4}{20}$, $\frac{3}{4}=\frac{15}{20}$. As $\frac{4}{20}$ is less than $\frac{15}{20}$, write $4\frac{4}{20}$ in the form $3\frac{24}{20}$. The problem is then $3\frac{24}{20}-2\frac{15}{20}=1\frac{9}{20}$; for $3-2=1$ and $\frac{24}{20}-\frac{15}{20}=\frac{9}{20}$.

11. Solve these problems:

(1) $13\frac{1}{8} - 7\frac{2}{5} = ?$ (3) $128\frac{4}{5} - 97\frac{15}{16} = ?$

(2) $30\frac{1}{20} - 16\frac{7}{8} = ?$ (4) $1217\frac{18}{61} - 989\frac{19}{25} = ?$

12. A tree $72\frac{3}{4}$ ft. high is broken off $28\frac{7}{15}$ ft. from the top. How high is the stump?

13. The 5-cent piece weighs $73\frac{1}{6}$ grains and the quarter-dollar $96\frac{9}{20}$ grains. How much heavier is the quarter than the 5-cent piece?

14. The dime weighs $38\frac{7}{12}$ grains. Before 1853 it weighed $41\frac{1}{4}$ grains. By how much was the weight of the dime reduced in 1853?

15. A man owned $\frac{3}{4}$ of a vessel and sold $\frac{2}{5}$ of the vessel. What part did he still own?

16. There were $3\frac{4}{5}$ acres in a tract of ground, and $1\frac{7}{8}$ acres were laid out in town lots. How much was not laid out in lots?

17. Write out rapidly the values of these sums and differences:

(1)	$\frac{1}{2} + \frac{1}{3} = ?$	(6)	$\frac{5}{8} - \frac{3}{5} = ?$	(11)	$\frac{6}{11} + \frac{2}{3} = ?$
(2)	$\frac{1}{4} - \frac{1}{5} = ?$	(7)	$\frac{4}{5} - \frac{1}{4} = ?$	(12)	$\frac{3}{8} - \frac{2}{9} = ?$
(3)	$\frac{2}{3} - \frac{1}{2} = ?$	(8)	$\frac{6}{7} + \frac{3}{4} = ?$	(13)	$\frac{4}{11} - \frac{3}{20} = ?$
(4)	$2\frac{3}{8} + 1\frac{5}{6} = ?$	(9)	$2\frac{8}{9} - 1\frac{3}{4} = ?$	(14)	$3\frac{6}{11} - 1\frac{3}{7} = ?$
(5)	$3\frac{1}{5} + 1\frac{3}{8} = ?$	(10)	$4\frac{3}{5} - 2\frac{1}{9} = ?$	(15)	$6\frac{7}{12} - 4\frac{2}{7} = ?$

18. The silver dollar weighs $412\frac{1}{2}$ grains and the double eagle weighs 516 grains. What is the difference between the weight of the double eagle and that of the silver dollar?

19. What is the difference between the weight of the half-dollar ($192\frac{9}{10}$ gr.) and that of the quarter-dollar ($96\frac{9}{20}$ gr.)?

20. In a sample of soil containing $46\frac{3}{8}$ ounces, $29\frac{5}{6}$ ounces was sandy loam. How many ounces of the soil was not sandy loam?

21. Six apples weighed $18\frac{3}{4}$ ounces before paring and coring, and $12\frac{3}{10}$ ounces after paring and coring. How many ounces were lost in paring and coring?

MULTIPLICATION OF FRACTIONS

§67. Multiplying a Fraction by a Whole Number.

1. 6 times 3 mi. = how many miles?

2. 8×4 yd. = how many yards?

3. 8×4 fifths = how many fifths?

4. $5 \times \frac{4}{7} = \frac{x}{7}$. What is the value of x?

5. $9 \times \frac{3}{19} = \frac{x}{19}$. What is the value of x?

6. A line is divided into 11 equal parts, and 5 of them are taken. What fraction represents the part of the whole which is taken?

7. What fraction would represent 3 times this part?

8. Replace the letter in each of these equations by its correct value. The sign ($\times$) should be read "times" in these problems.

(1) $6 \times \frac{2}{3} = \frac{x}{3}$ (5) $5 \times \frac{1}{6} = \frac{x}{6}$ (9) $2 \times \frac{3}{11} = \frac{x}{11}$

(2) $4 \times \frac{1}{2} = \frac{x}{2}$ (6) $3 \times \frac{5}{6} = \frac{x}{6}$ (10) $6 \times \frac{1}{7} = \frac{x}{7}$

(3) $3 \times \frac{1}{4} = \frac{x}{4}$ (7) $4 \times \frac{1}{9} = \frac{x}{9}$ (11) $5 \times \frac{2}{13} = \frac{x}{13}$

(4) $2 \times \frac{3}{4} = \frac{x}{4}$ (8) $2 \times \frac{4}{7} = \frac{x}{7}$ (12) $2 \times \frac{5}{12} = \frac{x}{12}$

In multiplying these fractions by the whole numbers what change was made in the numerator to get the product?

9. Make a rule for multiplying a fraction by a whole number.

10. Apply your rule to these problems, reducing to whole or mixed numbers all the products that are improper fractions:

(1) $3 \times \frac{4}{5} = ?$ (5) $11 \times \frac{3}{11} = ?$ (9) $6 \times \frac{7}{8} = ?$

(2) $5 \times \frac{3}{7} = ?$ (6) $9 \times \frac{1}{12} = ?$ (10) $8 \times \frac{5}{12} = ?$

(3) $7 \times \frac{2}{9} = ?$ (7) $4 \times \frac{5}{6} = ?$ (11) $10 \times \frac{5}{4} = ?$

(4) $9 \times \frac{5}{6} = ?$ (8) $6 \times \frac{4}{9} = ?$ (12) $9 \times \frac{11}{12} = ?$

NOTE.—In such exercises as (7) use cancellation, thus:

$$\overset{2}{\cancel{4}} \times \frac{5}{\cancel{6}_{3}} = \frac{10}{3} = 3\tfrac{1}{3}$$

11. Solve these problems and make a rule for multiplying a fraction quickly by its denominator:

(1) $6 \times \frac{1}{6} = ?$ (4) $7 \times \frac{5}{7} = ?$ (7) $21 \times \frac{13}{21} = ?$

(2) $9 \times \frac{1}{9} = ?$ (5) $12 \times \frac{5}{12} = ?$ (8) $48 \times \frac{135}{48} = ?$

(3) $5 \times \frac{3}{5} = ?$ (6) $18 \times \frac{12}{18} = ?$ (9) $75 \times \frac{46}{75} = ?$

NOTE.—In all such problems use cancellation, thus:

$$\cancel{7} \times \frac{5}{\cancel{7}} = 5$$

§68. Multiplying a Mixed Number by a Whole Number.

1. What will be the cost of 12 yd. of muslin at $8\frac{2}{3}$ cents?

SOLUTION: If 1 yd. costs $8\frac{2}{3}¢$ what will 12 yards cost? $12 \times 8\frac{2}{3}$ means $12 \times 8 + 12 \times \frac{2}{3} = 96 + 8 = 104$. *Ans.* $1.04.

In dealing with the expression $12 \times 8 + 12 \times \frac{2}{3}$, *or similar expressions, which contain both signs,* ($\times$) *and* ($+$), *the indicated multiplications should be performed first, and then the additions.*

2. $13 \times 4\frac{5}{9} = ?$

$4\frac{5}{9}$

13

$52 = 13 \times 4$

$7\frac{2}{9} = 13 \times \frac{5}{9}$

$59\frac{2}{9} = 13 \times 4\frac{5}{9}$

Multiply the whole numbers in the usual way.

$13 \times \frac{5}{9} = \frac{65}{9} = 7\frac{2}{9}$, as in problem 8, §67.

3. Solve the following, using the more convenient method in each case; employing cancellation whenever it shortens the work:

(1) $5 \times \frac{11}{15} = ?$ (5) $9 \times \frac{17}{36} = ?$ (9) $2 \times \frac{13}{4} = ?$
(2) $3 \times \frac{5}{9} = ?$ (6) $9 \times \frac{8}{9} = ?$ (10) $3 \times 6\frac{7}{8} = ?$
(3) $5 \times 4\frac{3}{4} = ?$ (7) $9 \times \frac{7}{12} = ?$ (11) $9 \times \frac{5}{12} = ?$
(4) $2 \times 2\frac{1}{2} = ?$ (8) $9 \times \frac{4}{5} = ?$ (12) $8 \times \frac{9}{11} = ?$

4. Find the cost of 25 yd. of cloth at $\$\frac{2}{5}$ per yard.

5. A earns $\$1\frac{3}{4}$, and B earns 4 times as much. B earns $\$x$. Find x.

6. A grocer sold $\frac{3}{4}$ bu. of apples to one customer, and 5 times that quantity to another. He sold y bu. to the second customer. Find y. If apples were \$.40 a peck what amount of money did the grocer receive from the first customer? From the second?

7. If a man can cut $\frac{7}{8}$ of a cord of wood in one day, how much can he cut in 6 da., working at the same rate?

8. One boy lives $\frac{5}{16}$ mi. from school and another 4 times as far. What is the distance from school to the second boy's home?

9. Solve problems based upon the following facts:

An iron tube weighs $\frac{7}{8}$ lb. per foot.

$\frac{8}{9}$ of the weight of water is oxygen.

The circumference of a bicycle wheel is $6\frac{17}{21}$ feet.

A west wind moved $47\frac{4}{5}$ mi. per hour.

§69. Multiplying a Whole Number by a Fraction.

1. $\frac{5}{7} \times 14 = ?$

SOLUTION: $\frac{5}{7} \times 14$ means $\frac{5}{7}$ of 14. $\frac{5}{7}$ of 14 means 5 of the $\frac{1}{7}$ths of 14. $\frac{1}{7}$ of $14 = 2$. $\frac{5}{7}$ of $14 = 5 \times 2 = 10$.

2. Solve these problems, reading the sign ($\times$) "times":

(1) $\frac{3}{5} \times 10 = ?$ (4) $\frac{2}{3} \times 15 = ?$ (7) $\frac{4}{9} \times 45 = ?$
(2) $\frac{4}{5} \times 25 = ?$ (5) $\frac{3}{7} \times 28 = ?$ (8) $\frac{8}{13} \times 26 = ?$
(3) $\frac{7}{8} \times 32 = ?$ (6) $\frac{6}{11} \times 33 = ?$ (9) $\frac{7}{11} \times 47 = ?$

SUGGESTION: In (9), $\frac{1}{11}$ of 47 = $4\frac{3}{11}$, and $7 \times 4\frac{3}{11} = 28\frac{21}{11} = 28 + 1\frac{10}{11} = 29\frac{10}{11}$. *Ans.* $29\frac{10}{11}$.

3. Solve the following:

(1) $\frac{9}{16} \times 50 = ?$ (3) $\frac{17}{25} \times 105 = ?$ (5) $\frac{5}{23} \times 110 = ?$

(2) $\frac{11}{18} \times 39 = ?$ (4) $\frac{25}{27} \times 220 = ?$ (6) $6\frac{7}{9} \times 28 = ?$

SUGGESTION: $6\frac{7}{9} \times 28$ means $6 \times 28 + \frac{7}{9} \times 28 = 168 + 21\frac{7}{9} = 189\frac{7}{9}$.

NOTE.—See statement in italics under problem 1, page 217.

§70. Factors May Be Interchanged.

The factors of a number are the numbers, which multiplied together, give the number.

The divisors of a number may also be called its factors.

Fig. 1 (*a*) represents a rectangle $\frac{1}{2}$ in. wide and 5 in. high. The area is then 5 times $\frac{1}{2}$ sq. in. = $2\frac{1}{2}$ square inches.

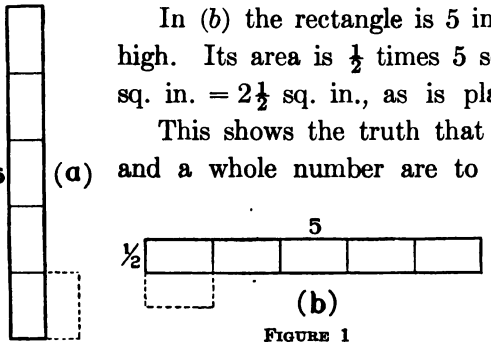


FIGURE 1

In (*b*) the rectangle is 5 in. long and $\frac{1}{2}$ in. high. Its area is $\frac{1}{2}$ times 5 sq. in., or $\frac{1}{2}$ of 5 sq. in. = $2\frac{1}{2}$ sq. in., as is plain from Fig. 1.

This shows the truth that when a fraction and a whole number are to be multiplied it makes no difference in the numerical value of the product which of the factors we regard as the multiplicand.

The *name* of the result is learned from the nature of the problem.

How may the product of a fraction by a whole number be readily found?

The product of a whole number by a fraction, or of a fraction by a whole number, equals the product of the whole number by the numerator, divided by the denominator.

1. Find the cost of $2\frac{1}{3}$ yd. of cloth at \$3 per yard.

2. A man had \$75; he spent $\frac{2}{3}$ of it for a bicycle, and $\frac{1}{5}$ of the remainder for clothing. Find the amount spent, and the amount he had left.

3. A gallon contains 231 cubic inches. How many cubic inches in $\frac{2}{3}$ gal.? In $\frac{5}{8}$ gallon?

4. A cubic foot of granite weighs 170 pounds. How many pounds in $5\frac{3}{4}$ cubic feet?

5. $\frac{7}{18}$ of a plot of ground containing 918 sq. rd. was fenced for a garden. The garden contained how many square rods?

6. Make and solve problems, based upon facts obtained by yourself, or upon those given below:

An avoirdupois pound of gold is worth about $\$348\frac{2}{5}$.

A tank contains 126 gal. linseed oil. Price per gallon, \$.62.

Corn was quoted and sold at $50\frac{7}{8}¢$ a bushel.

Wheat was quoted and sold at $77\frac{5}{8}¢$ a bushel.

Oats were quoted and sold at $32\frac{5}{8}¢$ a bushel.

Beans were quoted and sold at \$2.10 a bushel.

Apples were quoted and sold at $\$4\frac{1}{2}$ a barrel.

Potatoes were quoted and sold at $68\frac{3}{4}¢$ a bushel.

Calves were quoted and sold at $\$5\frac{3}{5}$ a hundredweight.

§71. Multiplying by Fractions and by Mixed Numbers.

(*A*) *Multiplying a fraction by a fraction.*

1. How many square feet are there in a rectangle $\frac{1}{2}' \times 8'$? $\frac{3}{4}' \times 12'$? $\frac{7}{8}' \times 16'$? $\frac{9}{10}' \times 30'$?

2. What is the area of a rectangle $\frac{2}{3}'' \times \frac{4}{5}''$? See Fig. 1.

SOLUTION: ABCD is a rectangle $\frac{4}{5}''$ long and $\frac{2}{3}''$ wide.

AEGH is a square inch.

What part of a square inch is ABKH? What part of ABKH is ABCD? Point out on the figure the part that represents $\frac{2}{3}$ of $\frac{4}{5}$ of 1 square inch?

Into how many small rectangles, such as a, do the dotted lines divide the square inch?

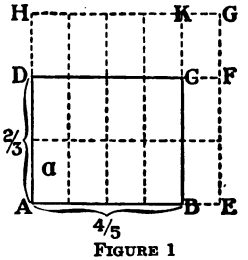


FIGURE 1

What part of the square inch is one of the small rectangles, a? How many of these are there in the given rectangle ABCD? The area of ABCD is then what part of a square inch?

We may write the two expressions for the area equal, thus:

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

Fractions may be multiplied by merely writing the product of the numerators over the product of the denominators with the dividing line between.

3. Draw a square inch and indicate within it the areas of the following rectangles:

(1) $\frac{2}{3}'' \times \frac{3}{4}'' = ?$ (3) $\frac{1}{9}'' \times \frac{5}{8}'' = ?$ (5) $\frac{7}{8}'' \times \frac{3}{4}'' = ?$

(2) $\frac{3}{4}'' \times \frac{2}{3}'' = ?$ (4) $\frac{3}{8}'' \times \frac{5}{6}'' = ?$ (6) $\frac{3}{4}'' \times \frac{7}{8}'' = ?$

4. Find the same products without the picture, using only the given numbers. Use cancellation.

5. By drawing and dividing a square inch, as in Fig. 1, show that the following are correct:

(1) $\frac{6}{7} \times \frac{5}{11} = \frac{6}{7}$ of $\frac{5}{11}$ (2) $\frac{5}{11} \times \frac{6}{7} = \frac{5}{11}$ of $\frac{6}{7}$ (3) $\frac{6}{7} \times \frac{5}{11} = \frac{30}{77}$

EXPLANATION: To find $\frac{1}{7}$ of $\frac{5}{11}$, find first $\frac{1}{7}$ of $\frac{1}{11}$.

Then find $5 \times \frac{1}{77} = \frac{5}{77}$.

But $\frac{6}{7}$ of $\frac{5}{11} = 6 \times \frac{1}{7}$ of $\frac{5}{11} = 6 \times \frac{5}{77} = \frac{30}{77}$.

As $\frac{6}{7} \times \frac{5}{11} = \frac{6}{7}$ of $\frac{5}{11}$, $\frac{6}{7} \times \frac{5}{11} = \frac{30}{77}$.

Similar reasoning would show also that $\frac{5}{11} \times \frac{6}{7} = \frac{30}{77}$. Now we

recall that the numerator (30) of the product ($\frac{5}{11} \times \frac{6}{7}$) was obtained by multiplying the numerators (6 and 5) of the factors ($\frac{5}{11}$ and $\frac{6}{7}$). How was the denominator (77) of the product obtained?

Always cancel common factors from numerators and denominators, when possible.

6. Solve *as rapidly as you can work accurately:*

(1) $\frac{3}{8} \times \frac{1}{2} = ?$ (4) $\frac{2}{5} \times \frac{1}{2} = ?$ (7) $\frac{2}{3} \times \frac{5}{8} = ?$

(2) $\frac{1}{2} \times \frac{3}{4} = ?$ (5) $\frac{3}{7} \times \frac{2}{3} = ?$ (8) $\frac{2}{7} \times \frac{3}{5} = ?$

(3) $\frac{3}{5} \times \frac{5}{9} = ?$ (6) $\frac{6}{7} \times \frac{3}{4} = ?$ (9) $\frac{5}{7} \times \frac{2}{9} = ?$

The product of any two fractions is a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators.

7. Answer as rapidly as you can correctly:

(1) $\frac{3}{4} \times \frac{3}{5} = ?$ (4) $\frac{3}{7} \times \frac{4}{5} = ?$ (7) $\frac{4}{7} \times \frac{6}{1} = ?$

(2) $\frac{4}{5} \times \frac{6}{7} = ?$ (5) $\frac{5}{8} \times \frac{3}{7} = ?$ (8) $\frac{7}{9} \times \frac{5}{1} = ?$

(3) $\frac{2}{3} \times \frac{4}{9} = ?$ (6) $\frac{2}{1} \times \frac{3}{5} = ?$ (9) $\frac{11}{12} \times \frac{7}{1} = ?$

8. Solve as rapidly as you can work correctly:

(1) $\frac{2}{3} \times \frac{6}{7} = ?$ (5) $\frac{2}{9} \times \frac{27}{32} = ?$ (9) $\frac{6}{7} \times \frac{56}{60} = ?$

(2) $\frac{3}{4} \times \frac{7}{9} = ?$ (6) $\frac{7}{8} \times \frac{8}{7} = ?$ (10) $\frac{10}{11} \times \frac{121}{130} = ?$

(3) $\frac{4}{5} \times \frac{25}{36} = ?$ (7) $\frac{8}{11} \times \frac{33}{40} = ?$ (11) $\frac{11}{12} \times \frac{132}{143} = ?$

(4) $\frac{3}{7} \times \frac{14}{27} = ?$ (8) $\frac{7}{12} \times \frac{60}{63} = ?$ (12) $\frac{13}{16} \times \frac{48}{65} = ?$

(B) Multiplying a mixed number by a mixed number.

9. $3\frac{2}{5} \times 5\frac{3}{4} = ?$

$3\frac{2}{5}$ times $5\frac{3}{4}$ means 3 times $5\frac{3}{4}$ and $\frac{2}{5}$ of $5\frac{3}{4}$.

The numbers may be multiplied in either of two ways:

thus, $5\frac{3}{4}$

$3\frac{2}{5}$

$15 = 3 \times 5$

$2\frac{1}{4} = 3 \times \frac{3}{4}$

$2 = \frac{2}{5}$ of 5

$\frac{6}{20} = \frac{2}{5}$ of $\frac{3}{4}$

$19\frac{11}{20}$

or thus, $3\frac{2}{5} = \frac{17}{5}$

$5\frac{3}{4} = \frac{23}{4}$

$3\frac{2}{5} \times 5\frac{3}{4} = \frac{17}{5} \times \frac{23}{4} = \frac{391}{20} = 19\frac{11}{20}$

When denominators of fractions divide evenly the whole numbers the fractions are to multiply the first way is usually better; otherwise, the second way is usually better.

10. Solve the first 6 of the following in one way and check the work the other way, solve the rest either way:

(1) $2\frac{1}{2} \times 3\frac{1}{3}$ (7) $3\frac{3}{5} \times 6\frac{2}{3}$ (13) $6\frac{7}{8} \times 22\frac{1}{2}$

(2) $2\frac{3}{4} \times 3\frac{1}{3}$ (8) $5\frac{3}{7} \times 5\frac{3}{7}$ (14) $12\frac{1}{2} \times 87\frac{1}{2}$

(3) $3\frac{1}{2} \times 3\frac{1}{2}$ (9) $5\frac{2}{5} \times 6\frac{2}{3}$ (15) $12\frac{1}{2} \times 37\frac{1}{2}$

(4) $4\frac{2}{3} \times 5\frac{1}{2}$ (10) $6\frac{3}{4} \times 8\frac{1}{3}$ (16) $37\frac{1}{2} \times 38\frac{7}{8}$

(5) $4\frac{2}{3} \times 4\frac{2}{3}$ (11) $25\frac{3}{4} \times 33\frac{1}{3}$ (17) $16\frac{5}{9} \times 81\frac{5}{8}$

(6) $4\frac{3}{4} \times 5\frac{2}{3}$ (12) $33\frac{1}{3} \times 66\frac{2}{3}$ (18) $80\frac{4}{11} \times 66\frac{3}{16}$

DIVISION OF FRACTIONS

§72. Dividing a Fraction by a Whole Number.

Do as much of this work orally as you can.

1. What is $\frac{1}{8}$ of 40 A.? 40 A. divided by 8 equals what?

2. $\frac{1}{7}$ of 35 lb. $=$? 35 lb. $\div$ 7 $=$?

3. $\frac{1}{10}$ of 80 $=$? 80 $\div$ 10 $=$?

4. $\frac{1}{3}$ of 3 fourths $=$? 3 fourths $\div$ 3 $=$?

5. $\frac{1}{5}$ of $\frac{5}{6}$ A. $=$? $\frac{5}{6}$ A. $\div$ 5 $=$?

6. $\frac{1}{3}$ of $\frac{6}{7}$ ft. $=$? $\frac{6}{7}$ ft. $\div$ 3 $=$?

Unit fractions, or fractional units are fractions whose numerators are 1.

7. Compare these unit fractions:

(1) $\frac{1}{4}$ is what part of $\frac{1}{2}$? (5) $\frac{1}{12}$ is what part of $\frac{1}{4}$?

(2) $\frac{1}{6}$ is what part of $\frac{1}{3}$? (6) $\frac{1}{15}$ is what part of $\frac{1}{5}$?

(3) $\frac{1}{8}$ is what part of $\frac{1}{4}$? (7) $\frac{1}{8}$ is what part of $\frac{1}{2}$?

(4) $\frac{1}{10}$ is what part of $\frac{1}{5}$? (8) $\frac{1}{12}$ is what part of $\frac{1}{3}$?

8. Divide $\frac{1}{2}$ by 2 $\frac{1}{3}$ by 2 $\frac{1}{4}$ by 2 $\frac{1}{5}$ by 2

9. Divide $\frac{1}{4}$ by 3 $\frac{1}{5}$ by 3 $\frac{1}{2}$ by 3 $\frac{1}{3}$ by 3

10. How is the size of a unit fraction changed by multiplying its denominator by 2? By 3? By 4? By 10? By 100? By a?

11. By what must the first fraction in each of the following pairs be multiplied to get the second?

(1) $\frac{2}{9}$ and $\frac{2}{3}$ (4) $\frac{8}{45}$ and $\frac{8}{9}$ (7) $\frac{37}{200}$ and $\frac{37}{50}$

(2) $\frac{3}{16}$ and $\frac{3}{4}$ (5) $\frac{13}{75}$ and $\frac{13}{15}$ (8) $\frac{a}{12b}$ and $\frac{a}{b}$

(3) $\frac{6}{14}$ and $\frac{6}{7}$ (6) $\frac{25}{96}$ and $\frac{25}{32}$ (9) $\frac{a}{12b}$ and $\frac{a}{12}$

12. Divide $\frac{2}{3}$ by 3 $\frac{3}{4}$ by 4 $\frac{6}{7}$ by 2 $\frac{8}{9}$ by 5 $\frac{13}{15}$ by 5 $\frac{25}{96}$ by 3

13. Make a rule for dividing a fraction by 3 without changing its numerator; by 5; by 28; by a.

Use pencil and paper only when needed.

14. What number should stand for x in these exercises?

(1) $\frac{72}{17} \div 3 = \frac{x}{17}$ (4) $\frac{256}{19} \div 16 = \frac{x}{19}$ (7) $\frac{22}{17} \div 9 = \frac{x}{17}$

(2) $\frac{75}{7} \div 15 = \frac{x}{7}$ (5) $\frac{125}{36} \div 25 = \frac{x}{36}$ (8) $\frac{ac}{b} \div a = \frac{x}{b}$

(3) $\frac{196}{15} \div 14 = \frac{x}{15}$ (6) $\frac{1728}{13} \div 144 = \frac{x}{13}$ (9) $\frac{mc}{n} \div c = \frac{x}{n}$

15. Solve, by finding what x stands for:

(1) $\frac{5}{6} \div 3 = \frac{5}{x}$ (3) $\frac{15}{16} \div 8 = \frac{15}{x}$ (5) $\frac{2411}{175} \div 3 = \frac{241}{x}$

(2) $\frac{7}{8} \div 4 = \frac{7}{x}$ (4) $\frac{13}{3} \div 25 = \frac{13}{x}$ (6) $\frac{9711}{341} \div 15 = \frac{971}{x}$

16. In what two ways may a fraction be divided by a whole number? When would you use the one method? The other?

A fraction may be divided by a whole number, (1) *by dividing its numerator by the whole number, without changing its denominator; or* (2) *by multiplying its denominator by the whole number, without changing its numerator.*

NOTE.—The usefulness of cancellation consists in making these two processes balance (undo) each other.

17. Divide $\frac{4}{3}$ lb. of shot equally among 5 boys, and give the weight of one share.

18. $\frac{7}{9}$ lb. of maple sugar is equally divided among 3 children. What part of a pound does each receive?

19. $\frac{4}{5}$ of a piece of work can be done in 8 days. What part of it can be done in 1 day?

20. Make and solve problems based upon the following items, showing work in each case:

$\$\frac{21}{25}$ to buy lace costing \$5 a yard

$\$\frac{11}{20}$ to buy apples costing \$3 per barrel

$\frac{3}{7}$ of an estate divided among 4 heirs

21. If $\frac{5}{8}$ lb. of coffee is divided into 5 equal portions, what part of a pound is there for each portion?

22. If 3 lb. of butter cost $\$\frac{3}{4}$, 1 lb. costs what part of \$1?

§73. Dividing a Mixed Number by a Whole Number.

1. 7 strips of carpeting cover a room $5\frac{1}{4}$ yd. wide. What is the width, in yards, of one strip?

$5\frac{1}{4}$ yd $= \frac{21}{4}$ yard.

$\frac{21}{4}$ yd. $\div 7 = \frac{3}{4}$ yd., the width of one strip.

$5\frac{1}{4}$ is what kind of number?

Before dividing what change was made? How was the division performed?

2. A boy earned $\$4\frac{1}{2}$ in 6 days. What part of a dollar did his earnings average per day?

$$4\tfrac{1}{2} = \tfrac{9}{2} \qquad \tfrac{9}{2} \div 6 = \frac{\cancel{9}^{3}}{2 \times \cancel{6}_{2}} = \tfrac{3}{4} \qquad \textit{Ans.}\ \tfrac{3}{4}.$$

How is the dividing done in this case?

3. $36\frac{2}{3}$ yd. of cloth were made up into 6 ladies' suits. If the suits contain the same number of yards, how many yards are there in each suit?

4. Make a rule for dividing a mixed by a whole number.

To divide a mixed number by a whole number first change the mixed number to an improper fraction and then proceed as in the division of a fraction by a whole number.

5. A ceiling 16 ft. wide contains $233\frac{1}{3}$ square feet. Find the length.

EXPLANATION: $233\frac{1}{3} = \frac{700}{3}$

$$\frac{700}{3} \div 16 = \frac{\overset{175}{\cancel{700}}}{3 \times \cancel{16}_{4}} = \frac{175}{12} = 14\frac{7}{12}$$

$14\frac{7}{12}$ ft. = 14′7″, the length of the room.

6. Solve the following:

(1) $14\frac{3}{4}$ bu. ÷ 8 =?
(2) $9\frac{7}{12}$ yd. ÷ 6 =?
(3) $16\frac{2}{3}$ hr. ÷ 12 =?
(4) $44\frac{9}{16}$ lb. ÷ 9 =?
(5) $39\frac{5}{8}$ mi. ÷ 7 =?
(6) $\$12\frac{5}{8}$ ÷ 5 =?
(7) $45\frac{9}{16}$ ÷ 5 =?
(8) $\frac{224}{5}$ ÷ 4 =?
(9) $14\frac{7}{21}$ ÷ 7 =?
(10) $66\frac{2}{3}$ ÷ 8 =?

7. Find the lengths of the surfaces with areas and widths as follows:

	Areas	Widths	Lengths
(1)	$212\frac{1}{2}$ sq. ft.	17 ft.	——
(2)	$261\frac{1}{2}$ sq. ft.	14 ft.	——
(3)	$406\frac{1}{4}$ sq. ft.	25 ft.	——
(4)	$164\frac{3}{4}$ sq. ft.	13 ft.	——
(5)	$457\frac{5}{7}$ sq. ft.	18 ft.	——
(6)	$202\frac{15}{16}$ sq. ft.	15 ft.	——

§74. Dividing by a Fraction.

1. How many hair ribbons $\frac{2}{3}$ yd. long can be made from 8 yd. of ribbon?

SOLUTION: How many thirds are there in 1? In 8?

$$\frac{\frac{24}{3}}{\frac{2}{3}} = \frac{24 \text{ thirds}}{2 \text{ thirds}} = \frac{24}{2} = 12 \quad \textit{Ans.}\ 12$$

2. How many $\frac{2}{5}$ths are there in $\frac{4}{5}$?

3. How many $\frac{3}{8}$ths are there in $\frac{9}{8}$?

4. How many $\frac{2}{3}$ds are there in $\frac{10}{3}$?

5. How many $\frac{2}{3}$ are there in $\frac{7}{8}$?

SOLUTION: The common unit for thirds and eighths is twenty-fourths.

$$\tfrac{2}{3} = \tfrac{16}{24} \quad \tfrac{7}{8} = \tfrac{21}{24}$$

$$\frac{\frac{7}{8}}{\frac{2}{3}} = \frac{\frac{21}{24}}{\frac{16}{24}} = \tfrac{21}{16} = 1\tfrac{5}{16} \quad \textit{Ans.}\ 1\tfrac{5}{16}.$$

We have seen that when the denominators are different, both fractions may be expressed in a common unit, and the numerators are then to be divided.

6. Answer the following:

(1) $\frac{3}{4} \div \frac{1}{4} = ?$ (7) $2 \div \frac{3}{4} = ?$ (13) $\frac{2}{3} \div \frac{3}{5} = ?$

(2) $\frac{7}{8} \div \frac{1}{8} = ?$ (8) $4 \div \frac{2}{5} = ?$ (14) $\frac{5}{7} \div \frac{3}{4} = ?$

(3) $\frac{6}{11} \div \frac{2}{11} = ?$ (9) $7 \div \frac{3}{8} = ?$ (15) $\frac{6}{11} \div \frac{5}{12} = ?$

(4) $\frac{10}{11} \div \frac{5}{11} = ?$ (10) $7 \div \frac{3}{7} = ?$ (16) $\frac{11}{12} \div \frac{7}{15} = ?$

(5) $\frac{11}{13} \div \frac{6}{13} = ?$ (11) $12 \div \frac{4}{5} = ?$ (17) $\frac{12}{17} \div \frac{9}{11} = ?$

(6) $\frac{15}{17} \div \frac{13}{17} = ?$ (12) $16 \div \frac{5}{9} = ?$ (18) $\frac{14}{15} \div \frac{12}{17} = ?$

§75. Dividing by Mixed Numbers.

1. Divide $3\frac{1}{2}$ by $2\frac{1}{3}$.

SOLUTION: $3\frac{1}{2} = \frac{7}{2}$; $2\frac{1}{3} = \frac{7}{3}$. Reduce both to the common unit, sixths.

$\frac{7}{2} = \frac{21}{6}$, and $\frac{7}{3} = \frac{14}{6}$.

$\frac{7}{2} \div \frac{7}{3} = \frac{21}{6} \div \frac{14}{6} = 21 \div 14 = 1\frac{7}{14} = 1\frac{1}{2}$.

2. Divide:

(1) $4\frac{1}{3}$ by $2\frac{1}{2}$ (4) 9 by $3\frac{1}{2}$ (7) $24\frac{3}{8}$ by $16\frac{2}{3}$

(2) $3\frac{3}{4}$ by $1\frac{2}{3}$ (5) $12\frac{1}{2}$ by $6\frac{1}{4}$ (8) 50 by $12\frac{1}{2}$

(3) $5\frac{3}{4}$ by $2\frac{3}{5}$ (6) $33\frac{1}{3}$ by $16\frac{2}{3}$ (9) 75 by $18\frac{3}{4}$

§76. Proportion.

1. Compare the ratio (or quotient) 3 : 2 with the ratio (or quotient) 6 : 4. What do you find?

2. Compare the ratio (or quotient) 8 da. : 2 da. with the ratio 16 men: 4 men.

3. Compare the ratio 10 ft. : 800 ft. with the ratio 100 mi. : 8000 miles.

A proportion is an equation of ratios. Thus 6 : 12 = 3 : 6, and $\frac{6}{12} = \frac{3}{6}$ ***are two different ways of writing the proportion.***

The first, second, third, and fourth numbers of the proportion are called the first, second, third, and fourth terms of the proportion.

The first and fourth terms are called the extremes, and the second and third terms, the means.

The first two terms are the first couplet, the third and fourth terms are the second couplet.

4. In 6 : 12 = 3 : 6, to what is the product of the extremes equal? The product of the means?

5. Answer the same questions for 3 : 5 = 12 : 20.

6. Which of these pairs of ratios may form proportions?

4 : 9 and 8 : 18 6 : 11 and 18 : 33 1 : 3 and 8 : 21

3 : 7 and 12 : 28 6 : 11 and 12 : 20 $\frac{3}{4}$ and $\frac{75}{100}$

7. Is $\frac{5}{7} = \frac{15}{21}$ a proportion?

Multiply both sides by 21 and we have $\frac{5}{7} \times 21 = 15$.

Now multiply both sides of this equation by 7 and we have $5 \times 21 = 7 \times 15$.

What are the terms 5 and 21 in the proportion called? What are the 7 and 15 in the proportion called?

8. Compare the product of the 1st and the 4th terms in these proportions with the product of the 2d and the 3d terms:

(1) $\frac{3}{7} = \frac{18}{42}$ (3) $\frac{12}{15} = \frac{36}{45}$

(2) $\frac{6}{13} = \frac{42}{91}$ (4) $\frac{9}{1} = \frac{108}{12}$

(5) $\frac{7}{11} = \frac{63}{99}$ (7) $\frac{a}{1} = \frac{18a}{18}$

(6) $\frac{2}{a} = \frac{24}{12a}$ (8) $\frac{x}{1} = \frac{20x}{x}$

State the principle problems 7 and 8 illustrate.

In a proportion the product of the means equals the product of the extremes.

9. What must x be in each of these expressions to give a proportion? Solution of first equation: $2x=15$, or $x=7.5$.

(1) $2:3 = 5:x$ (6) $a:b = 2a:x$

(2) $4:3 = 8:x$ (7) $18:x = 54:18$

(3) $6:x = 9:27$ (8) $4:6 = x:9$

(4) $7:2 = x:14$ (9) $9:x = 16:32$

(5) $x:6 = 8:12$ (10) $7:25 = 28:x$

§77. Practical Applications.

1. At the rate of 20 lb. for \$1.00, what will 15 lb. of sugar cost?

2. A boy buys 36 oranges at the rate of 3 for 5 cents. What do they all cost him?

NOTE.—Write the problem in the form $\frac{3}{5} = \frac{36}{c}$ and find c, the cost of the oranges.

3. The boy sells his 36 oranges at the rate of 2 for 5 cents. How much does he receive for them?

4. A man walks 54 miles at the rate of 9 mi. in 2 hours. How long does it take him to walk the 54 miles?

5. A number of boys buy 72 marbles at the rate of 6 marbles for 5 cents. How much do the 72 marbles cost?

6. If it takes 5 yd. of ribbon to trim 4 hats, how many hats can be trimmed with $37\frac{1}{2}$ yards?

7. At the rate of \$4.00 for 5 dozen, what will 3 doz. cocoanuts cost?

8. In a class there are 3 girls to every 4 boys. If there are 24 boys in the class, how many girls are there?

9. At the rate of \$7.00 for 3 days, how much money does a man earn in 21 days? In 27 days? In $16\frac{1}{2}$ days?

10. If it takes 7 yards of gingham to make 3 aprons, how many aprons will 42 yd. gingham make? 56 yd.? 63 yards?

11. If $\frac{7}{8}$ of a ton of coal is worth \$4.81, what are 12 T. worth at the same rate?

NOTE.—Call the unknown term x. Write the proportion, using x, and then use the principle on page 229, to find x.

12. A spelling class of 52 pupils writes a total of 1352 words; at the same rate how large a class will write a total of 728 words?

13. A girl jumping a rope makes 441 skips in 3 minutes. How long will it take her to make 392 skips at the same rate?

14. If the average column in a newspaper contains 1600 words, how much should a writer receive for 700 words at the rate of \$5 per column?

15. In going 10,725 ft. the front wheel of a bicycle revolves 1430 times. How far would it go in making 1001 revolutions?

DECIMAL FRACTIONS

§78. Notation of Decimals.

Answer all you can orally

1. In \$1111 what does the 1st 1 on the right stand for? The 2d? The 3d? The 4th?

2. In \$6666 what does the 1st 6 on the left denote? The 2d? The 3d? The 4th?

3. In \$372.68 what is the unit of the 3? (*Ans.* \$100) Of the 7? Of the 2? The 6? The 8?

4. In \$5555 how does the number denoted by each 5 compare with the number denoted by the 5 to its left? To its right?

5. How do the units of the places of a number change as we pass through the number from left to right?

6. Which unit is the fundamental unit out of which the other units are made? How is the place of this fundamental unit indicated in \$675.28?

A dot, called the decimal point, or point, is used to show the units' digit. The point always stands to the right of the units' digit or place.

7. If the law of problem 5 holds in 444.444, the unit of the 1st 4 to the right of the point equals what part of the unit of the 1st 4 to the left of the point? The unit of the 2d 4 to the right equals what part of the unit of the 1st 4 to the right?

8. In any number what part of the unit of the 1st place to the left of the point equals the unit of the 1st place to the right? Of the 2d place to the right? Of the 3d? Of the 4th?

The unit of the 1st place, or digit, to the right is called the tenth; of the 2d place, or digit, the hundredth; of the 3d, the thousandth; of the 4th, the ten-thousandth and so on.

§79. Numeration of Decimals.

1. The number 444.444 might be read, "4 hundreds, 4 tens, 4 units *and* 4 tenths, 4 hundredths, 4 thousandths"; but it is simpler to read it, ' Four hundred forty-four *and* four hundred forty-four *thousandths*." Read 234.234 both ways. Which is the shorter?

The latter mode of reading is used in practice. It may be stated thus: Read the integral (whole) part of the number, pronounce *and* for the decimal point, then read the part of the number to the right of the point as though it were a whole number and, finally, name the unit of the last digit on the extreme right.

2. Read 675.328; 236.89; 7.65; 43.6587.

3. Read the following:

(1)	.1	.01	.001	.0101	1.1	10.1
(2)	.6	6.06	66.6	.0606	600.06	6000.006
(3)	.60	7.070	8.50	.0600	87.087	10.0008

4. What is the ratio of 5 to .5? Of .5 to .05? Of .5 to .005? Of 55 to 5.5? Of 55 to .55?

5. Find the ratio of 875 to 87.5; of 87.5 to 8.75; of 875 to 8.75; of 875 to .875.

6. How is a number affected by moving the decimal point 1 place toward the left? (See problems 4 and 5.) 2 places? 3 places? 6 places?

7. What is a quick way of dividing any number by 10? By 100? By 1000?

8. Express the ratio of 2.358 to 23.58; to 235.8; to 2358; to 23,580; to .2358; to .02358.

QUERY.—Where is the point supposed to be in 2358? When the decimal point is not written, where is it supposed to be?

9. How is a number changed by moving the decimal point 1 place to the right? 2 places? 3 places? 4 places? 5 places? 6 places?

10. Make a rule for quickly multiplying any number by 10; by 100; by 1000; by 1 followed by any number of zeros.

11. What effect is produced in such a number as .683 or .492 by writing a zero between the decimal point and its first digit? 2 zeros? Any number of zeros?

12. Compare .5 with each of these numbers: .50; .500; .50000. What is the effect of writing any number of zeros to the right of the last digit of a decimal?

A decimal fraction, or decimal, is a fraction whose denominator is 10, 100, 1000, or some power of ten, in which the denominator is not written but is indicated by the position of the decimal point.

A power of 10 is a number obtained by using 10 as a factor any number of times. Thus, $10 \times 10 = 10^2 = 100$; $10 \times 10 \times 10 = 10^3 = 1000$, and so on, are powers of 10.

§80. To Reduce a Decimal to a Common Fraction.

The fractions we have studied whose denominators are actually written, are called *common fractions.*

1. Express the following decimals as common fractions:

.5 .50 .500 .4 .40 .400 .6 .60 .25 .250 .125 .375 .625 .875

2. Reduce the results of problem 1 to their lowest terms.

3. Write these mixed decimals as improper fractions:

1.5 2.50 2.75 10.4 12.5 6.75 18.25

4. Reduce the results of problem 3 to their lowest terms.

5. After dropping the decimal point from the following decimals, what numbers must be written beneath them to express them as common fractions?

.6 .67 .625 .875 .1275 12.75 25.786 33.333 6.6666

6. Make a rule for expressing any decimal as a common fraction in its lowest terms.

§81. Addition.

RAINFALL, INDUSTRIAL PROBLEMS, ETC.

See definition of rainfall, page 152.

1. The following quantities of rain fell from week to week during May, 1902, in Chicago: First week, 1.09 in.; second week, 1.11 in.; third week, .45 in., and fourth week, 2.43 inches. Find the total rainfall for the month.

CONVENIENT FORM

1.09 in.
1.11 in.
.45 in.
2.43 in.
Ans. 5.08 in.

EXPLANATION: It is convenient to write the addends in a column so that the unit digits are all in the same vertical column. Then begin on the right and add as with whole numbers. In the sum the point should stand directly under the points in the addends.

Finding the sum of decimal numbers is called addition of decimals.

2. A coal dealer received 8 carloads of coal of the following tonnages: 24.6, 28.785, 31.25, 24.95, 31.8, 25.125, 28, and 29.25. What was the total tonnage (number of tons)?

3. A farm was divided by its owner into lots of the following acreages (number of acres): $32.87\frac{1}{2}$, $7.12\frac{1}{2}$, $68.33\frac{1}{3}$, $11.66\frac{2}{3}$, $16.28\frac{3}{8}$, and $21.13\frac{7}{8}$. What was the total area of the farm?

4. The following numbers represent in thousands of feet a lumber dealer's sales in 1 da.: 6.865, 24.245, 16.398, 12.28, 18.2, 6.395, 24, and 18.967. What were the total sales for the day?

5. A quantity of soil contained .125 lb. gravel; .268 lb. coarse sand; .175 lb. fine sand; $.037\frac{1}{2}$ lb. organic matter; $.214\frac{3}{4}$ lb. clay, and .275 lb. water. What was the total weight of the soil?

6. During 8 hr. a freight train made the following mileages: 32.15, 28.375, 15.687, 20.2, 15.63, 17.5, 8.95, and 21.3. How far did the train run during the 8 hours?

7. Following are the weights in grains of the U. S. coins: 1¢-piece, 48; 5¢-piece, 73.166⅔; dime, 38.583⅓; quarter-dollar, 96.45; half-dollar, 192.9; dollar, 412½; quarter-eagle, 64½; half-eagle, 129; eagle, 258; double eagle, 516. Find the weight of all together.

§82. Subtraction.

NATURE STUDY

1. 54 cu. in. of soil in its natural state weighed 1.94 pounds. After being thoroughly dried it weighed 1.459 pounds. How much moisture passed off in drying?

CONVENIENT FORM

1.940 lb.
1.459 lb.
Ans. .481 lb.

EXPLANATION: We have seen that 1.94 may be written 1.940. For convenience write the numbers so that units' digits stand in the same column. Beginning on the right subtract as though the numbers were whole numbers. In the result the point should stand directly under the points in the minuend and subtrahend.

Finding the difference of decimal numbers is called subtraction of decimals.

2. After drying, the same soil occupied only 37.125 cu. in. How much did it shrink in bulk in drying?

3. 100 green oak leaves weighed .22 pounds. After thorough drying they weighed .087 pounds. What was the weight of water contained in the green leaves?

4. The organic matter was then driven off by burning the dry leaves. The ash weighed .0053 pounds. How much organic matter did the 100 leaves contain?

5. The corresponding numbers for 100 green elm leave were: Weight of green leaves, .132 pounds; weight of d·

leaves, .0345 pounds; weight of ash, .0035 pounds. Answer questions like 3 and 4 for these leaves.

6. 1.135 lb. of dry beans, soaked for 24 hr., weighed 2.212 pounds. What was the weight of water taken up by the beans?

7. The dry beans occupied 34.875 cu. in. and the soaked beans 85.5 cubic inches. Find the increase in bulk.

§83. Stature and Weight of Persons.

The following table contains the average heights and weights of boys, girls, men, and women for the ages indicated by the numbers in the first column. Heights are given in feet, and weights in pounds:

Age Yr.	Heights		Diff.	Growth in Height		Weights		Diff.	Growth in Weight	
	Boys and Men	Girls and Women		Boys and Men	Girls and Women	Boys and Men	Girls and Women		Boys and Men	Girls and Women
2	2.60	2.56				25.01	23.53			
4	3.04	3.00				31.38	28.67			
6	3.44	3.38				38.80	35.29			
9	4.00	3.92				49.95	47.10			
11	4.36	4.26				59.77	56.57			
13	4.72	4.60				75.81	72.65			
15	5.07	4.92				96.40	89.04			
17	5.36	5.10				116.56	104.43			
18	5.44	5.13				127.59	112.55			
20	5.49	5.16				132.46	115.30			
30	5.52	5.18				140.38	119.82			
40	5.52	5.18				140.42	121.81			
50	5.49	5.04				139.96	123.86			
60	5.38	4.97				136.07	119.76			
70	5.32	4.97				131.27	113.60			
80	5.29	4.94				127.54	108.80			
90	5.29	4.94				127.54	108.81			

1. Fill out, on a separate slip, the columns headed difference (Diff.); the first by subtracting the height of the

girls and women from that of the boys and men of the same age, and the second by subtracting the weights of girls and women from those of boys and men of the same age. Subtract without rewriting the numbers.

2. There are 4 vacant columns headed Growth; two for growth in height, and two for growth in weight. Fill out, on a separate slip, columns like the first of these from the column of heights of boys and men by subtracting each number of this column from the one next below it. Tell what the difference means.

3. Fill out similarly a column like the second from the column of heights of girls and women.

4. In a similar way fill out the last two columns from the columns of weights of boys and men and weights of girls and women. Tell what these differences mean.

5. At what age are boys growing most rapidly in height? In weight? At what age do men begin to decrease in height? In weight?

6. Answer questions similar to 4 for girls and women.

7. Compare your own height and weight with the numbers of this table, for your age.

NOTE.—If your age is not in the table, it will be between two ages given there. Use the mean of the numbers for these two ages for your comparison.

§84. Multiplication of Decimals. Pointing the Product.

Answer all you can orally.

1. What is the relation of the numbers in the following pairs?

(1) 25 and 2.5	(4) 1.28 and 12.8	(7) 2847 and 28.47
(2) 25 and .25	(5) 1.28 and 128	(8) 284.7 and 2.847
(3) 75 and 7.5	(6) 47.8 and 4.78	(9) 28.47 and .2847

2. The product 37×25 equals how many times the product 37×2.5?

3. The product 684×7.5 equals what part of the product 684×75?

4. What is the product 684×75? What, then, is the product 684×7.5? What is the product $684 \times .75$? $684 \times .075$? $68.4 \times .075$? $6.84 \times .075$? $.684 \times .075$?

By the number of decimal places of a number is meant the number of digits (zero included) on the right of the decimal point.

ILLUSTRATION: In 2.005 there are 3 decimal places.

Use pencil and paper only when needed.

5. Find the product 4862×784, and from it write the following products:

486.2×784	48.62×784	48.62×78.4
4.862×78.4	4.862×7.84	$4862 \times .784$

6. How many decimal places are there in 486.2? In 48.62? 4.862? 10.03? 3.0060? .0600? 20.0806?

7. How many decimal places are there in each of the products of problem 5?

8. Compare the number of decimal places in each of the products of problem 5 with the sum of the numbers of decimal places in both the multiplicand and the multiplier. What do you find?

9. Make a rule for finding how many decimal places there must be in the product of two decimals.

10. How, then, can you find where the decimal point belongs in the product of any two decimals?

The number of decimal places in the product equals the sum of the numbers of decimal places in the factors.

11. A field containing 38.75 A. yielded 23.9 bu. of wheat per acre. What was the total yield?

SOLUTION: Since each acre yielded 23.9 bu., 38.75 A. must yield 38.75×23.9 bushels.

CONVENIENT FORM	
With Integers	With Decimals
3875	38.75
239	23.9
34875	34875
11625	11625
7750	7750
926125	926.125

EXPLANATION: Multiply as with whole numbers, then point the product so as to make as many decimal figures in it as there are in both factors.

Ans. 926.125 bu.

12. A steer weighing 16.22 cwt. sold at \$8.85 per hundredweight. What price did it bring?

13. A passenger train ran for 12.27 hr. at the rate of 65.75 mi. per hour. How far did it run during the time?

§85. Division by an Integer.

1. I paid \$941.25 for 15 A. of land; what was the price per acre?

CONVENIENT FORM	
With Integers	With Decimals
6275	62.75
15)94125	15)941.25
90	90
41	41.
30	30.
112	11.2
105	10.5
75	.75
75	.75

EXPLANATION: Compare the steps in the work with decimals with the corresponding steps in the work with integers.

How do the decimal points stand through the problem? How does the point stand in the quotient?

Check: $15 \times 62.75 = 941.25$.

Ans. \$62.75.

2. I paid \$2823.75 for 45 A. of land. What was the price paid per acre?

§86. Dividing by a Decimal.

Answer all you can orally.

1. How do the following numbers compare in size: 4320, 432, 43.2, 4.32, and .432?

2. Where is the decimal point supposed to be in 36? (See query p. 232.) In 125? In 6? In 12? In 6829?

3. Read the following horizontally:

(1)	$366 \div 6 = 61$	$36.6 \div 6 = 6.1$	$.366 \div 6 = .061$
(2)	$496 \div 8 = 62$	$49.6 \div 8 = 6.2$	$.496 \div 6 = .062$
(3)	$242 \div 11 = 22$	$24.2 \div 11 = 2.2$	$.242 \div 11 = .022$

How is the quotient changed in value by keeping a fixed divisor and moving the decimal point in the dividend toward the left 1 place? 2 places? 3 places? 4 places?

How is the quotient affected by keeping a fixed divisor and changing the decimal point in the dividend toward the right 1 place? 2 places? 3 places? 4 places?

4. Notice the dividend, the divisor, and the quotient in the following:

10	100	1000	10000
$1000\overline{)10000}$	$100\overline{)10000}$	$10\overline{)10000}$	$1\overline{)10000}$
4	40	400	4000
$1111\overline{)4444}$	$111.1\overline{)4444}$	$11.11\overline{)4444}$	$1.111\overline{)4444}$
2	20	200	2000
$303\overline{)606}$	$30.3\overline{)606}$	$3.03\overline{)606}$	$.303\overline{)606}$

How does keeping the same dividend and moving the decimal point in the divisor toward the left 1 place change the quotient? 2 places? 3 places? 4 places?

With the same dividend what effect would be had on the quotient by moving the decimal point toward the right in the divisor 1 place? 2 places? 3 places? 4 places?

$\frac{360}{120}$ is another way of writing "Divide 360 by 120."

5. Answer the following questions:

$\frac{360}{120} = ?$ $\frac{36}{12} = ?$ $\frac{3600}{1200} = ?$ $\frac{3.6}{1.2} = ?$

6. How, then, does it affect the quotient to move the decimal point toward the left, or toward the right in both dividend and divisor 1 place? 2 places? 3 places? 4 places?

Hence, to point the quotient in division of decimals: (1) *divide through as though both dividend and divisor were whole numbers;* (2) *move the point in both dividend and divisor toward the right,* or the left, until there is but one figure to the left of the point in the divisor. Then determine *by estimate,* about what the quotient must be, and point accordingly.

§87. Exercises and Problems.

1. Divide 78.975 by 32.5.

```
      243
   ------
325)78975
   650
   ----
   1397
   1300
   ----
    975
    975
   ----
```

EXPLANATION.—The figures of the quotient are 243.

Imagine the points in 78.975 and 32.5 both moved 1 place to the left, and the divisor becomes 3.25, and the dividend 7.8975.

Disregard the decimals, and point the 243 so as to get as nearly as possible $\frac{1}{3}$ of 7, something near 2. The quotient is 2.43.

2. Solve the following:

(1) $89.76 \div 26.4 = ?$
(2) $89.6002 \div 24.82 = ?$
(3) $3.234 \div 24.5 = ?$
(4) $135.88 \div 21.5 = ?$
(5) $3.234 \div .132 = ?$
(6) $9.456 \div .24 = ?$
(7) $524.660 \div 28.36 = ?$
(8) $135.88 \div 6.32 = ?$
(9) $9.456 \div 39.4 = ?$
(10) $409.625 \div 56.5 = ?$

3. Check the results of problem 2 by multiplying the quotients by the divisors and comparing the products with the dividends.

No.	Weight	Price
1	1503	$9.00
2	1622	8.85
3	1606	8.60
4	1524	8.50
5	1524	8.50
6	1504	8.10
7	936	8.70
8	1327	6.85
9	1273	8.05
10	1130	8.75
11	1326	7.65
12	1141	8.50
Total..		
Aver...		

4. On Dec. 6, 1901, a herd of 12 prize cattle of weights given in the table sold at the indicated prices per cwt. What was the total weight of the herd? The average weight of the animals?

5. For how much did the entire herd sell?

6. What was the average price per hundredweight?

7. What was the average price per head which the owner received for the herd?

8. The previous year on the same occasion, a carload of 26 prize-winning Angus cattle, averaging 1492 lb., sold at $15.50 per hundredweight. What average price did the cattle bring the owner?

9. How much did he receive for the carload of problem 8?

The following table gives, for men, of ages from 18 to 26 years, the average lung capacity in cubic inches, the height in inches, and the weight in pounds.

Age	Lung Capacity	Height	Weight
18	251.4	68.2	134.25
19	251.8	68.2	135.40
20	258.2	68.1	138.65
21	260.4	68.1	140.60
22	264.8	68.2	141.15
23	263.7	68.1	138.60
24	267.1	68.2	143.90
25	267.2	68.2	143.15
26	267.1	68.9	142.30

10. Find the number of cubic inches of lung capacity per inch of height, for each age, by computing (finding) the ratio to 2 decimal places of each number of column 2 to the corresponding number of column 3. Is this ratio the same for all ages?

11. Similarly find the number of cubic inches, to 2 decimal places, of lung capacity per pound of weight, for each age.

12. For each age find the number of pounds of weight per inch of height to 2 decimal places. Are these numbers the same for all ages?

§88. Original Problems.

Make and solve problems based on the following facts:

1. 1 cu. ft. is about .8 of a bushel of small grain. A grain bin is $8' \times 12' \times 22'$.

2. 1 bu. of ear corn is about 2.25 cubic feet. A wagon box is $2.67' \times 2.85' \times 9.6'$.

3. Well settled timothy hay runs about 355.25 cu. ft. to the ton. A hay shed is $24' \times 40'$ and is filled with settled hay to a height of 18.5′.

4. Loose timothy hay runs about 460 cu. ft. to the ton. A load of loose hay is $8' \times 18' \times 22.7'$.

5. Stove coal runs about 35.1 cu. ft. to the ton. A coal bin is $6' \times 12.5' \times 15.35'$.

6. 1 perch of stone is 24.75 cubic feet. A stone wall is $2.75' \times 4.385' \times 126.8'$.

7. 30.25 sq. yd. make 1 sq. rd. A town lot is 60.5 yd. $\times$ 42.36 yards.

8. 272.25 sq. ft. make 1 sq. rd. A field is 408.375 ft. $\times$ 1361.25 feet.

§89. To Reduce a Common Fraction to a Decimal.

1. Express $\frac{5}{16}$ decimally.

$$
\begin{array}{r|l}
16 & 5.0000 \\
\hline
 & .3125 \\
 & 4.8 \\
 & \overline{.20} \\
 & .16 \\
 & \overline{40} \\
 & 32 \\
 & \overline{80} \\
 & 80 \\
 & \overline{}
\end{array}
$$

EXPLANATION.—Annex zeros to the right of the decimal point after the numerator, and then divide by the denominator.

Ans. $\frac{5}{16} = .3125$

2. Express the following common fractions decimally (to 3 decimal places):

$\frac{3}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{7}{8}$ $\frac{5}{8}$ $\frac{3}{16}$ $\frac{5}{32}$ $\frac{14}{64}$

3. Express the following mixed numbers decimally (to 3 places):

$1\frac{1}{2}$ $3\frac{1}{5}$ $2\frac{3}{4}$ $6\frac{5}{8}$ $13\frac{5}{16}$ $18\frac{13}{32}$

4. Express the following decimally to 5 decimal places:

$\frac{1}{7}$ $\frac{3}{13}$ $\frac{8}{17}$ $\frac{9}{23}$ $\frac{16}{37}$ $\frac{5}{19}$

Such decimals as these fractions give rise to, that do not terminate, are called non-terminating decimals.

5. Express the following numbers decimally to 6 decimal places:

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{9}$ $\frac{2}{9}$ $\frac{7}{9}$ $\frac{4}{11}$ $\frac{6}{11}$ $\frac{9}{11}$ $\frac{10}{27}$ $\frac{11}{27}$ $\frac{20}{27}$

Such non-terminating decimals as these that repeat the same digit or group of digits indefinitely, are called repetends, or circulating decimals, or circulates.

6. Express the values of these numbers by the use of integers and decimals only:

$1.3\frac{1}{2}$ $16.87\frac{1}{2}$ $5.28\frac{3}{5}$ $17.9\frac{5}{8}$ $20.0\frac{2}{5}$ $1.00\frac{4}{5}$

$.00\frac{7}{8}$ $.0\frac{7}{8}$ $.000\frac{1}{16}$ $30.060\frac{3}{5}$

7. Express the following fractions decimally to 4 places:

$\frac{11}{12}$ $\frac{10}{13}$ $\frac{7}{19}$ $\frac{3}{11}$ $4\frac{2}{9}$ $\frac{3\frac{1}{2}}{7\frac{1}{2}}$ $\frac{6.7}{7.8}$ $\frac{8.65}{12.17}$ $\frac{7\frac{3}{5}}{12\frac{7}{9}}$

8. A man sold $37\frac{1}{2}$ A. of his farm of $79\frac{3}{4}$ acres. How many hundredths of his farm did he sell?

9. A man owned a piece of city land 450′ square. A strip $37\frac{1}{2}$′ wide was cut from each of its four sides for streets. How many 100ths of the square were cut away?

10. How many hundredths of the area of a page of this book are in the margins? (Measure to the nearest 16th of an inch).

11. How many hundredths of the area of the surface of your desk is the area of the surface of your book?

12. How many hundredths of the area of the surface of the floor is the area of the surface of your desk?

PERCENTAGE

§90. Measuring by Hundredths.

Answer all you can orally.

1. What is $\frac{1}{100}$ of 500 miles? $\frac{3}{100}$ of 500 miles? $\frac{8}{100}$ of 500 miles?

2. What is $\frac{3}{100}$ of 200 acres? $\frac{5}{100}$ of 200 acres? $\frac{12}{100}$ of 200 acres?

3. What is $\frac{50}{100}$ of 100 feet? $\frac{37}{100}$ of 100 feet? $\frac{33\frac{1}{3}}{100}$ of 100 feet?

4. $\frac{1}{2}$ of anything equals how many hundredths of it?

5. What is $\frac{1}{4}$ of 20? What is 25% of 20?

6. What is $\frac{3}{4}$ of 24? What is 75% of 24?

7. What is $\frac{1}{5}$ of 30? 20% of 30? $\frac{2}{5}$ of 30? 40% of 30?

8. What is $\frac{3}{5}$ of 30? 60% of 30? $\frac{4}{5}$ of 30? 80% of 30?

9. What is $\frac{1}{3}$ of 18? $33\frac{1}{3}$% of 18? $\frac{2}{3}$ of 18? $66\frac{2}{3}$% of 18?

10. What is $\frac{1}{10}$ of 40? 10% of 40? $\frac{3}{10}$ of 40? 30% of 40? $\frac{7}{10}$ of 40? 70% of 40?

11. What is $\frac{1}{20}$ of 80? 5% of 80? $\frac{3}{20}$ of 80? 15% of 80? $\frac{7}{20}$ of 80? 35% of 80?

12. What is $\frac{1}{25}$ of 50? 4% of 50? $\frac{4}{25}$ of 50? 16% of 50? $\frac{8}{25}$ of 50? 32% of 50?

13. What is $\frac{1}{50}$ of 100? 2% of 100? $\frac{3}{50}$ of 100? 6% of 100?

14. What is $\frac{1}{100}$ of 200? 1% of 200? $\frac{3}{100}$ of 200? 3% of 200?

15. How many hundredths of anything are the following fractional parts of it?

$\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$ $\frac{1}{20}$ $\frac{1}{25}$ $\frac{3}{20}$ $\frac{8}{25}$

One hundredth is often written 1% and read 1 percent. The sign (%) is a short way of writing "one one-hundredth." Percent is a short way of saying "one one-hundredth."

16. Read and give the meaning of the following:

2% 6% 8% 12% $12\frac{1}{2}$% 25% $33\frac{1}{3}$% $87\frac{1}{2}$% 100%

17. Express the following decimally to 3 places:

$\frac{5}{8}$ $\frac{17}{20}$ $\frac{11}{12}$ $3\frac{5}{9}$ $5\frac{7}{32}$ $\frac{11}{12\frac{1}{2}}$ $\frac{2\frac{1}{2}}{5\frac{1}{2}}$ $\frac{4.5}{5.6}$

§91. Decimals as percents.

Answer all you can orally.

1. What is .10 of 20? What is 10% of 20?

2. What is .20 of 20? What is 20% of 20?

3. What is .40 of 10? What is 40% of 10?

4. What is .60 of 40? What is 60% of 40?

5. What is .25 of 100? What is 25% of 100?

6. What is .50 of 80? What is 50% of 80?

7. What is .25 of 200? What is 25% of 200?

8. What is .01 of 100? Of 200? Of 500? Of 800?

9. What is 1% of 100? Of 200? Of 500? Of 800?

10. What is .02 of 100? Of 200? Of 500? Of 150?

11. What is 2% of 100? Of 200? Of 500? Of 150?

12. What is .05 of 200? Of 300? Of 800?

13. What is 5% of 200? Of 300? Of 800?

14. Write in percents the equivalents of the following decimals:

.15 .25 .75 .18 .36 .12$\frac{1}{2}$ .33$\frac{1}{3}$ .66$\frac{2}{3}$ .37$\frac{1}{2}$ 1.13

15. Write in the form of decimal fractions the equivalents of the following percents:

6% 10% 15% 16% 20% 37% 37$\frac{1}{2}$% 66$\frac{2}{3}$%

16. From the foregoing examples tell how to change decimals to percents; percents to decimals.

17. Compare your statements with—

To change decimals to percents move the decimal point two places to the right and write % after the number.

18. Tell how to change percents to decimal fractions.

19. What is .01 of 80? .02 of 80? .03 of 80? .10 of 80?

20. What is 1% of 80? 2% of 80? 3% of 80? 10% of 80?

21. What is .12 of 100? .12 of 200? .12 of 500?

22. What is 12% of 100? 12% of 200? 12% of 500?

23. What is .12$\frac{1}{2}$ of 200? .12$\frac{1}{2}$ of 300? .12$\frac{1}{2}$ of 500?

24. What is .125 of 200? .125 of 300? .125 of 500?

25. What is 12$\frac{1}{2}$% of 200? 12$\frac{1}{2}$% of 300? 12$\frac{1}{2}$% of 500?

26. What is 12.5% of 200? 12.5% of 300? Of 500?

§92. Equivalent parts, decimals and percents.

Answer all you can orally.

1. What is $\frac{1}{4}$ of 12? .25 of 12? 25% of 12?

2. What is $\frac{3}{4}$ of 12? .75 of 12? 75% of 12?

3. What is $\frac{1}{2}$ of 10? .50 of 10? 50% of 10?

4. What is $\frac{1}{3}$ of 15? .33$\frac{1}{3}$ of 15? 33$\frac{1}{3}$% of 15?

5. What is $\frac{2}{3}$ of 15? .66$\frac{2}{3}$ of 15? 66$\frac{2}{3}$% of 15?
6. What is $\frac{1}{5}$ of 30? .20 of 30? 20% of 30?
7. What is $\frac{2}{5}$ of 30? .40 of 30? 40% of 30?
8. What is $\frac{3}{5}$ of 30? .60 of 30? 60% of 30?
9. What is $\frac{4}{5}$ of 20? .80 of 20? 80% of 20?
10. What is $\frac{1}{8}$ of 16? .12$\frac{1}{2}$ of 16? 12$\frac{1}{2}$% of 16?
11. What is $\frac{3}{8}$ of 16? .37$\frac{1}{2}$ of 16? 37$\frac{1}{2}$% of 16?
12. What is $\frac{5}{8}$ of 16? .62$\frac{1}{2}$ of 16? 62$\frac{1}{2}$% of 16?
13. What is $\frac{7}{8}$ of 16? .87$\frac{1}{2}$ of 16? 87$\frac{1}{2}$% of 16?
14. What is $\frac{1}{10}$ of 40? .10 of 40? 10% of 40?
15. What is $\frac{3}{10}$ of 40? .30 of 40? 30% of 40?
16. What is $\frac{7}{10}$ of 40? .70 of 40? 70% of 40?
17. Express in three ways: $\frac{1}{4}$ of 12; $\frac{3}{4}$ of 12; $\frac{1}{10}$ of 20; .2 of 25; .5 of 7; 25% of 6; 33$\frac{1}{3}$% of 21; .125 of 16.
18. What is $\frac{1}{8}$ of 24? .125 of 24? 12.5% of 24?
19. What is $\frac{3}{8}$ of 16? .375 of 16? 37.5% of 16?
20. What is $\frac{5}{8}$ of 32? .625 of 32? 62.5% of 32?
21. What is $\frac{1}{16}$ of 32? .06$\frac{1}{4}$ of 32? 6$\frac{1}{4}$% of 32?
22. What is $\frac{3}{16}$ of 48? .18$\frac{3}{4}$ of 48? 18$\frac{3}{4}$% of 48?
23. What is $\frac{11}{50}$ of 50? .22 of 50? 22% of 50?
24. What is $\frac{37}{100}$ of 200? .37 of 200? 37% of 200?

§93. Finding the percentage.

Answer all you can orally.

1. What is 50% of 4? Of 12? Of 18? Of 24? Of 36?
2. What is 25% of 8? Of 12? Of 20? Of 32? Of 44?
3. What is 20% of 5? Of 15? Of 20? Of 35?
4. What is 33$\frac{1}{3}$% of 3? Of 15? Of 21? Of 27?
5. What is 10% of 20? Of 60? Of 80? Of 90?
6. What is 4% of 25? Of 50? Of 100? Of 75?
7. What is 5% of 20? Of 40? Of 60? Of 80?

8. How many pounds are 50% of 8 pounds? Of 18 pounds? Of 24 pounds?

9. How many square feet are 25% of 16 square feet? Of 48 square feet? Of 88 square feet? Of 400 square feet?

10. A man set out 150 fruit-trees and 28% of them died. How many died?

Finding the percentage is the same as finding the product in multiplication of decimals. [$150 \times .28 = (?)$]

150 *base* (multiplicand)
.28 *rate* (multiplier)
1200
300
42.00 *percentage* (product)

The number on which the percentage is to be found is the base.

The rate is the percent to be found.

The percentage is a certain number of hundredths of the base.

IN DECIMALS { multiplicand corresponds to *base*; multiplier corresponds to *rate*; product corresponds to *percentage* } IN PERCENTAGE

11. Find the percentage in the following exercises:

(1) 12% of 35	(7) 75 % of 650
(2) 18% of 65	(8) 95 % of 1200
(3) 30% of 68	(9) 85 % of 1000
(4) 45% of 90	(10) 85 % of 150
(5) 24% of 45	(11) $25\frac{1}{2}$% of 600
(6) 36% of 80	(12) $37\frac{1}{2}$% of 800

12. Heating an iron rod 100 in. long increased its length 1%. How many inches was the length increased?

13. One boy threw a stone 100 ft., and another threw it 12% farther. How many feet farther did the second boy throw the stone?

14. Referring to Fig. 2 (page 131), point out the following percents of it: 20%; 40%; 2%; 4%; 10%; 100%.

15. Draw a square and divide it by a line so as to show 50% of it; 25% of it; 75% of it; $12\frac{1}{2}\%$ of it.

16. Show the same percents using a circle; rectangle.

17. Draw a line of any length and show $33\frac{1}{3}\%$ of it; $66\frac{2}{3}\%$ of it; 100% of it; $12\frac{1}{2}\%$ of it; 10% of it; 20% of it.

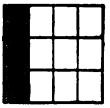


FIGURE 1

18. Draw a square and divide it into small rectangles as shown in the figure. Point out the following percents of it:

25% $33\frac{1}{3}\%$ 50% $66\frac{2}{3}\%$ 75% 100%.

19. A pair of shoes costing \$2 was sold at a gain of 50%. What was the amount gained?

20. A boy was 50 in. in height 2 years ago. Since then he has grown 4% higher. How many inches taller is he now than he was two years ago? How tall is he now?

21. A boy runs $12\frac{1}{2}\%$ of the distance he goes on an errand that requires him to go 240 yards and to return the same distance. How far does he run?

22. A girl's flower-bed is 80% of the size of her brother's garden bed. The garden bed is 10 ft. by 12 ft. How many square feet does her flower-bed contain?

23. Two boys ran a 2-minute race. One ran 160 rd. the other 5% farther. How far did the second boy run?

24. A boy on a bicycle raced with a street-car while it ran 350 rd., and was beaten by 4% of this distance. By how many rods was he beaten?

§94. Finding the percent or rate.

Answer all you can orally.

1. 4 is what percent of 8? Of 16? Of 20? Of 12?

2. 6 is what percent of 12? Of 24? Of 18? Of 30?

3. 10 is what percent of 20? Of 30? Of 40? Of 50?

4. 12 is what percent of 24? Of 36? Of 48? Of 60?

5. 24 is what percent of 72? Of 96? Of 120?

6. The base is 320 and the percentage is 48. Find the rate.

Finding the rate in Percentage is the same as finding the multiplier in Decimals. [$320 \times (?) = 48.00$]

.15 *rate*

base 320)48.00 *percentage*

320

1600

1600

7. Find the rate in the following exercises:

(1) base, 240, percentage, 72
(2) base, 120, percentage, 6
(3) base, 65, percentage, 13
(4) base, 96, percentage, 24
(5) base, 104, percentage, 26
(6) base, 144, percentage, 48
(7) base, 160, percentage, 80
(8) base, 240, percentage, 80

8. Out of 80 games played by a championship team, 20 were lost. What percent of the games were lost? What percent were won?

9. A man paid \$18 for the use of \$300 for a year. What percent of the sum borrowed was the sum paid?

10. A boy's grade in October in arithmetic was 60 and in November it was 80. By what percent of his grade for October did he increase it in November?

11. The enrollment of a class was 40 and 8 pupils were absent. What percent of the class was absent?

12. On a certain day there were 14 hours from sunrise to sunset. It rained $3\frac{1}{2}$ hours of this time. What percent of the 14 hours was rainy?

13. If school begins at 9 A. M. and closes at 4 P. M. with an hour recess for dinner, what percent of a 24-hour day does school last?

14. A girl goes to bed at 9 P. M. and rises at 6 A. M. What percent of a 24-hour day is she in bed?

§95. Finding the Base.

Answer all you can orally.

1. 50% of a number is 2. What is the number?

2. 25% of a number is 3. What is the number?

3. 20% of a number is 6. What is the number?

4. 75% of a number is 9. What is the number?

5. 40% of a number is 10. What is the number?

6. 18% of the number of apple-trees in an orchard died. 36 trees died. How many trees were in the orchard?

Finding the base in Percentage is the same as finding the multiplicand in Decimals. [(?) $\times .18 = 36.00$]

$$\begin{array}{l} \textit{rate}.18\overline{)36.00} \quad \textit{percentage} \\ \quad\;\; 200 \quad \textit{base} \end{array}$$

7. Find the *base* in the following exercises:

(1) percent, .20, percentage, 44	(6) percent, .16, percentage, 48
(2) percent, .25, percentage, 64	(7) percent, .12, percentage, 84
(3) percent, .75, percentage, 120	(8) percent, .28, percentage, 140
(4) percent, .64, percentage, 128	(9) percent, .45, percentage, 180
(5) percent, .27, percentage, 81	(10) percent, .06, percentage, 126

8. Fifteen percent of the distance a boy can jump is 12 inches. How far can the boy jump?

9. A flock of pigeons increased 24, which is an increase of 60%. How many were in the flock.

10. Twenty percent of the cost of raising a potato-crop was $12. What was the whole cost?

11. A girl spent 25 cents which was $12\frac{1}{2}\%$ of her monthly allowance. How much was her allowance?

12. A boy spends $33\frac{1}{3}\%$ of his study-time on his arithmetic lesson. He spends $\frac{3}{4}$ of an hour on his arithmetic lesson. How much time does he study?

13. A certain school has 180 school days in a school year, which is $12\frac{1}{2}\%$ of the number of school days of the elementary school. How many school days has the elementary school?

14. A boy went to school 16 years which was 20% of the age of his grandfather. How old was his grandfather?

§96. Easy Applications of Percentage.

1. The height of a 14 year old boy is 60 in., and a girl of the same age is 2% taller. How tall is the girl?

2. A dress cost the dealer \$2.00 and was sold for $33\frac{1}{3}\%$ more than it cost. For how much did it sell?

3. The price of an article was reduced \$1.00, which was 20% of the marked price. Find the marked price.

4. Eighty percent of the cost of a girl's suit is \$4.00. What is 1% of the cost? What is 100%, the whole cost?

5. A thermometer reading was 120°, which was 20% higher than the reading taken 10 min. before. What was the previous reading?

NOTE.—In such questions as this answer first the question, "20% of what?" In this case 20% of the previous reading.

6. This year a flat rented for \$55 a month, which is 10% more than it rented for last year. For how much did it rent last year?

7. In 25 lb. of macaroni there are 2.5 lb. of water. What percent of the macaroni is water?

8. In 1902 there were 6500 acres sown to sugar beets in the state of New York. In 1903 there were 7150 acres

in the same state. What was the percent of increase from 1902 to 1903?

9. There were 3400 acres of sugar beets in Wisconsin in 1902, and in 1903, there were 70% more acres. How many acres of sugar beets were there in Wisconsin in 1903?

10. Michigan raised 150 acres of cranberries in 1900 and Maine raised 60% of this number of acres. How many acres of cranberries did Maine raise?

11. Rhode Island had 300 acres of cranberries in 1900 and Connecticut had 90% of this number of acres. How many acres had Connecticut?

12. A young poplar tree increased in height by 120% during 1908. At the beginning of the year it was 10 ft. tall. How tall was it at the end of the year?

13. Joseph bought 21 young chickens and rats killed 7 of them the first night he owned them. What percent of the chickens were killed? What percent remained alive?

14. A boy paid 15 cents apiece for 24 young Plymouth Rock chickens. He spent 40 cents for feed, raised 8 of them and sold the 8 at 60 cents each. What was his percent of gain, or loss?

15. A girl earned 90 cents picking berries and spent 40% of her earnings. How many cents had she left?

16. The living expenses of a family for a year are $640. The cost of clothing is $128. What percent of the living expenses is the cost of clothing?

17. A fruit farm has 5 acres in apples, which is 40% of the whole farm. How many acres are in the farm?

18. A peach orchard occupies 12% of a 12½-acre farm. How many acres are there in the peach orchard?

19. Fifty-five percent of the metal of a watch case,

weighing 2 oz., is gold. What is the weight of gold in the watch case?

20. A man paid \$6 for the use of \$100 for one year. What percent was what he paid of what he borrowed?

21. At the same rate as in problem 20 what should a man pay for the use of \$200 for one year?

22. At the rate in problem 20 what should the man pay for the use of \$100 for 2 yr.? For 3 yr.? For 5 years?

23. How much must a man pay, at the rate of \$4 on the \$100, for the use of \$3200 for 1 yr.? For 2 years?

INTEREST

§97. Simple Interest.

An extensive use of the system of measurement by hundredths is made with problems in Interest.

Interest is money to be paid for the use of money.

1. Two percent interest means that 2¢ is to be paid every year for the use of 100¢ or \$1, \$2 for the use of \$100, and so on at this rate. What then would 6% interest mean? 4% interest? 10% interest?

The rate of interest is the number of dollars paid for every \$100 borrowed for 1 year.

In problem 1 above, the rates are 2 percent, 6 percent, 4 percent, and 10 percent, and they are written 2%, 6%, 4%, and 10%.

2. A man pays \$10 for the use of \$200 for 1 year. What is the rate of interest?

3. What is the rate of interest if \$30 is paid for the use of \$500 for 1 year?

4. What are the rates of interest in the following?

(1) \$15 paid for the use of \$300 for 1 year
(2) \$20 for the use of \$400 for 1 year
(3) \$24 for the use of \$400 for 1 year
(4) \$36 for the use of \$600 for 1 year
(5) \$32 for the use of \$400 for 1 year
(6) \$42 for the use of \$700 for 1 year

The interest for one year is the sum of money to be paid each year for the whole sum borrowed.

5. What is the interest for 1 year on \$200 at 6%? At 5%?

6. What is the interest for 1 year on \$400 at 4%? At 7%?

7. What is the interest for 1 year on the following?

(1) \$500 at 6%
(2) \$400 at 5%
(3) \$700 at 6%
(4) \$800 at 4%
(5) \$800 at 7%
(6) \$600 at 5%
(7) \$350 at 6%
(8) \$560 at 4%

Reckoning interest for 1 year is the same thing as reckoning percentage; thus—

PERCENTAGE	INTEREST FOR ONE YEAR
\$800 *base*	\$800 *principal*
05 *rate*	.05 *rate*
\$40.00 *percentage*	\$40.00 *interest for* 1 *year*

The only things to be carefully noted are—

(1) that two of the numbers are named differently in interest-problems; and

(2) that the interest for any number of years is the interest for 1 year multiplied by the number of years.

Thus, if the interest for 1 year is \$40, as above, the interest for 4 years is 4 times \$40, and for $5\frac{1}{2}$ years is $5\frac{1}{2} \times$ \$40, etc.

8. Find the interest on \$200 at 4% for 1 yr. 6 years.

9. What is the interest at 6% on \$1 for 1 year?

NOTE.—In computing interest a year means 12 months of 30 days each.

10. What is the interest at 6% on \$1 for $\frac{1}{2}$ year? For 6 mo.? For 2 mo.? For 1 month?

11. What is the interest at 6% on \$1 for 3 months? For 5 months? For 7 months? For 14 months? For 16 months? For 34 months? For any number of months? For n months?

12. From the answer to problem 9, find the interest at 6% for 1 yr. on \$8; on \$25; on \$48; on \$85; on \$124.

13. If you know the interest at 6% for 1 yr. on \$1, how can you find the interest at 6% for 1 yr. on any number of dollars?

14. From the last answer to problem 12, find the interest at 6% on \$124 for 2 years; for 3 years; for $4\frac{1}{2}$ years; for $12\frac{1}{2}$ years; for any number of years.

INTEREST, WHEN THE TIME IS IN YEARS AND MONTHS

15. If the interest on a sum of money at 4% for 1 year is known, how would you find the interest on the same sum at 4% for $\frac{1}{12}$ of a year, or 1 month? For 2 months? For 4 months? For 3 months? For 5 months? For 7 months?

16. The interest on \$40 at 6% for 1 year is \$2.40. Find the interest at 6% for $\frac{1}{12}$ of a year, or 1 month; for 2 months; for 4 months; for 3 months; for 7 months.

17. Tell how to find the interest at 6% on any number of dollars for any number of months.

18. Find the interest on \$60 at 6% for 1 year and 1 month; for 1 yr. and 2 months; for 1 yr. and 3 months; for 1 yr. 5 mo.; for 1 yr. 7 months.

19. Find the interest on \$80 at 5% for 2 years; for 2 yr. 1 mo.; for 2 yr. 3 mo.; for 3 yr. 6 mo.; for 2 yr. 7 mo.; for 3 yr. 5 months.

20. Find the interest on \$1200 at 4% for 2 yr. and 6 mo.; for 2 yr. 9 mo.; for 2 yr. 3 mo.; for 3 yr. 3 mo.; for 3 yr. 8 months.

INTEREST RECKONED ON THE 6%-BASIS

21. What part of the interest on any sum at 6% is the interest on the same sum for the same time at 1%? At 3%? At 2%? At 4%?

22. The interest on a sum of money for 1 yr. at 6% is \$24. What is the interest on the same sum for the same time at 3%? At 2%? At 12%? At 18%? At 1%?

23. The interest on a sum of money for a certain time at 6% is \$24, and on the same sum for the same time at 1% it is \$4. What is the interest on the same sum for the same time at 7%? At 5%? At 8%? At 4%? At 9%? At 3%?

24. The interest at 6% on a certain sum for a certain time is \$36. Find the interest on the same sum for the same time at the following rates: 1%, 7%, 5%, $\frac{1}{2}$%, $6\frac{1}{2}$%, $5\frac{1}{2}$%, 3%, $3\frac{1}{2}$%, $7\frac{1}{2}$%, $\frac{1}{4}$%, $6\frac{1}{4}$%, $5\frac{3}{4}$%.

25. By finding the interest first at 6%, then taking $\frac{1}{6}$ of it, find the interest at 7% on \$800 for 3 years.

26. By finding the interest first at 6%, find the interest for $4\frac{1}{2}$ years at 5% on \$560.

27. Find the interest at 9% on \$720 for 3 yr. by the 6%-method.

CALCULATING FIRST THE INTEREST ON \$1

Some prefer to calculate interest by finding first the interest on \$1 for the given time and at the given rate.

Answer all you can orally.

28. What is the interest on \$1 for 1 year at 6%? At 4%? At 7%? At 8%? At 3%? At 1%? At 9%? At 10%?

29. What is the interest on \$1 for $\frac{1}{2}$ year at 6%? At 4%? At 7%? At 8%? At 3%? At 1%? At 9%? At 10%?

30. What is the interest for 3 months on \$1 at 6%? At 4%? At 8%? At 7%? At 5%?

31. Find the interest on \$1 at 6% for 2 yr.; for 6 mo.; for 1 yr. 6 mo.; for 2 mo.; for 2 yr. 2 mo.; for 2 yr. 6 months.

32. Find the interest on \$1 at 8% for 6 mo.; for 3 mo.; for 9 mo.; for 1 yr. 3 mo.; for 1 yr. 9 mo.; for 2 yr. 6 mo.; for 2 yr. 9 mo.; for 3 yr. 9 months.

33. From the results of problem 32 find the interest for 9 mo. at 8% on \$5; on \$10; on \$20; on \$50; on \$100.

34. Tell how to find the interest on any sum by finding first the interest on \$1 for the given time and rate.

35. The interest on \$1 for a given time and rate is \$.15. Find the interest for the same time and rate on \$50; on \$150; on \$450; on \$865; on \$1260.

To find the interest by the $1*-method on any sum of money multiply the interest on* $1 *for the given time and at the given rate, by the number of dollars.*

36. Find by the $1-method the interest at 6% on $320 for 2 yr. 6 months.

37. Find the interest on $350 at 4% for 2 yr. 6 mo.; for 6 yr. 9 mo., for 8 yr. 3 months by the $1-method.

38. Find by the $1-method the interest on—

(1) $500 at 7% for 1 yr. 3 mo.
(2) $460 at 8% for 1 yr. 9 mo.
(3) $620 at 5% for 4 yr. 4 mo.
(4) $150 at $8\frac{1}{2}$% for 3 yr. 6 mo.
(5) $810 at $4\frac{1}{2}$% for 4 yr. 4 mo.

39. Find by the $1-method and check by the 6%-method the interest on—

(1) $650 at 7% for 2 yr. 9 mo.
(2) $800 at 5% for 5 yr. 6 mo.
(3) $1250 at $4\frac{1}{2}$% for 4 yr. 3 mo.
(4) $1560 at 4 % for 6 yr. 6 mo.

40. Find by any method the interest on the following:

(1) $210 at 8% for 1 yr. 6 mo.
(2) $360 at 3% for 2 yr. 3 mo.
(3) $450 at 4% for 3 yr. 9 mo.
(4) $600 at 7% for 1 yr. 5 mo.
(5) $720 at 5% for 3 yr. 7 mo.
(6) $1224 at 4% for 2 yr. 11 mo.

MULTIPLICATION TABLES

1× 1= 1	2× 1= 2	3× 1= 3	4× 1= 4
1× 2= 2	2× 2= 4	3× 2= 6	4× 2= 8
1× 3= 3	2× 3= 6	3× 3= 9	4× 3=12
1× 4= 4	2× 4= 8	3× 4=12	4× 4=16
1× 5= 5	2× 5=10	3× 5=15	4× 5=20
1× 6= 6	2× 6=12	3× 6=18	4× 6=24
1× 7= 7	2× 7=14	3× 7=21	4× 7=28
1× 8= 8	2× 8=16	3× 8=24	4× 8=32
1× 9= 9	2× 9=18	3× 9=27	4× 9=36
1×10=10	2×10=20	3×10=30	4×10=40
1×11=11	2×11=22	3×11=33	4×11=44
1×12=12	2×12=24	3×12=36	4×12=48
5× 1= 5	6× 1= 6	7× 1= 7	8× 1= 8
5× 2=10	6× 2=12	7× 2=14	8× 2=16
5× 3=15	6× 3=18	7× 3=21	8× 3=24
5× 4=20	6× 4=24	7× 4=28	8× 4=32
5× 5=25	6× 5=30	7× 5=35	8× 5=40
5× 6=30	6× 6=36	7× 6=42	8× 6=48
5× 7=35	6× 7=42	7× 7=49	8× 7=56
5× 8=40	6× 8=48	7× 8=56	8× 8=64
5× 9=45	6× 9=54	7× 9=63	8× 9=72
5×10=50	6×10=60	7×10=70	8×10=80
5×11=55	6×11=66	7×11=77	8×11=88
5×12=60	6×12=72	7×12=84	8×12=96
9× 1= 9	10× 1= 10	11× 1= 11	12× 1= 12
9× 2= 18	10× 2= 20	11× 2= 22	12× 2= 24
9× 3= 27	10× 3= 30	11× 3= 33	12× 3= 36
9× 4= 36	10× 4= 40	11× 4= 44	12× 4= 48
9× 5= 45	10× 5= 50	11× 5= 55	12× 5= 60
9× 6= 54	10× 6= 60	11× 6= 66	12× 6= 72
9× 7= 63	10× 7= 70	11× 7= 77	12× 7= 84
9× 8= 72	10× 8= 80	11× 8= 88	12× 8= 96
9× 9= 81	10× 9= 90	11× 9= 99	12× 9=108
9×10= 90	10×10=100	11×10=110	12×10=120
9×11= 99	10×11=110	11×11=121	12×11=132
9×12=108	10×12=120	11×12=132	12×12=144

TABLES OF WEIGHTS AND MEASURES

LIQUID MEASURE

4 gills (gi.)	= 1 pint	(pt.)
2 pints	= 1 quart	(qt.)
4 quarts	= 1 gallon	(gal.)
1 gallon	= 231 cubic inches	
31½ gallons	= 1 barrel	(bbl.)

DRY MEASURE

2 pints (pt.)	= 1 quart	(qt.)
8 quarts	= 1 peck	(pk.)
4 pecks	= 1 bushel	(bu.)
1 bushel	= 2150.42 cubic inches	

AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound	(lb.)
2000 pounds	= 1 ton	(T.)

LINEAR MEASURE

12 inches (in.)	= 1 foot	(ft.)
3 feet	= 1 yard	(yd.)
5½ yards, or 16½ feet	= 1 rod	(rd.)
320 rods, or 5280 feet	= 1 mile	(m.)

SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot	(sq. ft.)
9 square feet	= 1 square yard	(sq. yd.)
30¼ square yards, or 272¼ square feet	= 1 square rod	(sq. rd.)
160 square rods	= 1 acre	(A.)
640 acres	= 1 square mile	(sq. m.)

SOLID OR CUBIC MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot	(cu. ft.)
27 cubic feet	= 1 cubic yard	(cu. yd.)

WOOD MEASURE

16 cubic feet	= 1 cord foot	(cd. ft.)
8 cord feet, or 128 cubic feet	= 1 cord	(cd.)

TIME MEASURE

60 seconds (sec.)	= 1 minute	(min.)
60 minutes	= 1 hour	(hr.)
24 hours	= 1 day	(da.)
7 days	= 1 week	(wk.)
365 days	= 1 common year	(c. yr.)
366 days	= 1 leap year	(l. yr.)
100 years	= 1 century	(C.)

MISCELLANEOUS TABLES

12 units	= 1 dozen
12 dozen	= 1 gross
20 units	= 1 score
24 sheets	= 1 quire
20 quires	= 1 ream

SUMMARY FOR WORK OF PART I

I. What the pupil should know at the end of this part.

1. In Notation and Numeration:

(1) How to read and to write nine-digit whole numbers.
(2) How to read and to write common fractions with one- and two-digit terms.
(3) How to read, and to write decimal numbers to hundredths.
(4) How to write, and to interpret numbers in Roman numerals.
(5) How to write and to read denominate numbers involving two or three denominate units.

2. In Addition and Subtraction:

(1) How to analyze and to solve problems on motion involving two or more operations with whole and simple fractional numbers.
(2) How to add and to subtract whole numbers, common fractions with one- and two-digit numerators, and decimal numbers to hundredths.
(3) How to add and to subtract denominate numbers, involving three denominate units or less.
(4) How to foot and to balance simple bills.
(5) How to check addition by adding in a changed order, and how to check subtraction by addition.
(6) The terminology of addition and subtraction.

3. In Multiplication and Division:

(1) How to multiply all sorts of whole numbers and how to divide whole numbers with one-, two-, and three-digit divisors.
(2) How to multiply and to divide such fractional and decimal numbers as he can read.
(3) How to multiply and to divide denominate numbers by whole numbers.
(4) How to obtain areas from dimensions, and dimensions from areas, etc., of rectangles, parallelograms, and triangles.
(5) How to multiply mentally by numbers larger than 12 and how to recognize readily factors to 10 of two- or three-digit composite numbers.
(6) How to multiply by 10, or 100, or 1000, by shifting the decimal point.

(7) How to check division by multiplication.
(8) The terminology of multiplication and division.
(9) How to recognize factors and products of multiplication table quickly.

4. In Common Fractions:

(1) How to add, subtract, multiply, and divide simple fractional values of distances, areas, etc.
(2) How to handle readily the numbers $12\frac{1}{2}$ and $16\frac{2}{3}$ as parts of 100, etc., in many practical uses and applications.
(3) How to picture and how to figure without the picture with fractions with one- and two-digit terms.
(4) Necessary parts of the terminology of common fractions.
(5) How to use cancellation intelligently in multiplying and reducing fractions.

5. In Decimal Fractions:

(1) How to read, write, add, subtract, multiply, and divide decimals to hundredths.
(2) How to express easier fractions and percents as decimals, and *vice versa*.
(3) How to place the point intelligently in sums, differences, products, and quotients with decimals containing one or two places.
(4) How to multiply or divide by 10, or 100, or 1000, by shifting the decimal point.

6. In Mensuration and Scale Drawing:

(1) How to make and to read scale drawings.
(2) Areas of rectangles, parallelograms, and triangles in sq. ft., sq. yd., sq. rd., and sq. mi.
(3) How to apply simple scales to drawings to ascertain actual dimensions, areas, etc., of the objects represented.
(4) How to find unknown parts of areas, objects, etc., from data given in denominate units.
(5) How to use the square and rectangle to picture operations with fractional numbers.

7. In Percentage:

(1) How to read and to write percents.
(2) How to indicate percents, and to express simpler percents in their fractional or decimal equivalents.

(3) How to find simple percents of magnitudes and of numbers, as so many 100ths of the magnitude or number.
(4) How to picture certain percents of squares, rectangles, and circles.
(5) How to find the *base*, and how to find the *rate*, in easy exercises.
(6) Necessary terms of percentage.

8. In Compound Numbers.

(1) How to write, read, add, subtract, multiply, divide, and reduce easier denominate numbers, expressed in three units, or less, in problems in distance, area, capacity, bulk, weight, time.
(2) About how large, how heavy, etc., the denominate units are.

II. Ideas and Values of arithmetical study advanced by the work of Part I.

1. Power to know when to add, subtract, multiply, or divide.
2. Power to estimate quickly what results will be.
3. Power to apply the arithmetical operations to the better understanding or appreciation of lists of interesting and useful number facts.
4. Ability to recognize the call for arithmetic in practical affairs.
5. Increased power of self-reliance engendered through habitual checking and proving of result.
6. Feeling of growing power and skill engendered through well-timed drill exercises.
7. Gradual increase in number insight fostered through the habit of "answering all you can orally," of "using the pencil only when needed."
8. Heightening the interest through the continual interplay of the insight, developed through drill exercises, and the conscious feeling of power to do things worth while, developed through problems dealing with the appreciably real.
9. Fostering the disposition to understand, and to challenge what is hazy, or doubtful, by continual insistence upon reasons, checks, and proofs of results and work.
10. Strengthening the power of concentration through brief topical studies, and through single pages now and then, of unified problems, dealing with out-of-school interests that call for all the arithmetical operations.

SUMMARY FOR WORK OF PART II

I. What the pupil should know at the end of this part.

1. In Notation and Numeration:

(1) Digits, defined and taught.
(2) Name and place values of digits, taught and applied.
(3) The Periods, units, thousands, millions, taught and applied.
(4) Decimal notation carried to hundredths.
(5) Arabic and Roman notation and numerals.
(6) How to change from Roman to Arabic numerals, and *vice versa*.

2. In Addition and Subtraction:

(1) Meaning of addition, and of subtraction, and of the symbols: +, −, and =.
(2) Definition of addition, sum, addends, minuend, subtrahend, difference, or remainder, and literal numbers.
(3) When to add, and when to subtract.
(4) How to add and subtract (a) with all sorts of whole numbers, and to check or test work, (b) with common fractions having one, two, and three digit terms, and (c) with decimals to thousandths.
(5) How to find and to express sums and differences of easy literal numbers.

3. In Multiplication and Division:

(1) The meaning of multiplication, of division, and of the symbols: ×, $\underline{)\quad}$, $\overline{)\quad}$, ⅜ and 3/7.
(2) Definitions of multiplicand, multiplier, product, factor, prime and composite numbers, dividend, divisor, quotient, remainder (in division).
(3) When to multiply, and when to divide.
(4) Methods of shortening multiplications and divisions.
(5) Tests of divisibility, by 2, 3, 4, 5, 8, 9, 10, etc.
(6) Both short and long division.
(7) Checking multiplication and division, and cancellation.

(8) How to multiply and divide (a) with all sorts of integers, (b) with common fractions of one, two, and three digit terms, and (c) with decimals to thousandths.
(9) How to find and to express products and quotients of easy literal numbers.

4. In Bills and Accounts:

(1) How to calculate wages from time-sheet.
(2) How to draw bills for purchases and sales, and how to balance and receipt them.
(3) How to keep family expense accounts.
(4) Informal use of Debits and Credits, as Receipts and Expenditures, respectively.

5. On the Equation:

(1) Meaning of equation (1) as a statement of a number relation, and (2) as an expression of balance of values.
(2) How to find x in such equations as $7x + 8 = 29$, or $22x - 6 = 38$, etc.
(3) Meanings of first member, or side; second member, or side.
(4) Proportion defined and used as an equation of ratios.

6. Ratio and Proportion:

(1) Ratio defined as a quotient of numbers and as the result of measuring numbers.
(2) Proportion defined as an equation of ratios.

7. In Common Fractions:

(1) Definition of a fraction (a) as one or more equal parts of a whole; (b) as a quotient. Definition of numerator, denominator, and terms.
(2) How to express an integer, or mixed number fractionally, and *vice versa*.
(3) How to reduce fractions to higher, lower, and lowest terms.
(4) How to add and subtract fractions both graphically and arithmetically, (a) when denominators are the same number; (b) when one denominator is a simple multiple of the other; (c) when denominators are different.
(5) How to multiply and to divide a whole number by fractions.

(6) How to multiply and to divide a fraction by a whole number, or by a fraction.
(7) How to reduce common to decimal fractions.
(8) The factors in fractional products may be interchanged.

8. In Decimal Fractions:

(1) Meaning and use of the decimal point.
(2) Reading and writing decimal numbers taught as a continuation of the decimal notation.
(2) Reduction of common to decimal fractions.
(3) Addition and subtraction of decimal numbers.
(4) Pointing the product and the quotient in decimals.
(5) Meaning of repetends and circulates.
(6) Reduction of decimals to common fractions.

9. In Percentage:

(1) How to express common and decimal fractions as percents, and *vice versa*.
(2) Percentage as a system of reckoning by hundredths.
(3) How to find the most frequently occurring percents of smaller numbers.

10. In Interest:

(1) Meaning of simple interest.
(2) How to calculate simple problems in interest for times given in years.
(3) Relation of interest at 6% with interest at other rates.

11. In Scale Drawing and Mensuration:

(1) Making drawings to scales of moderate difficulty.
(2) Reading and interpreting scale drawings.
(3) Scale drawings, and graphs of thermometer readings, and of statistical data.
(4) Drawing developments of regular rooms.
(5) The laws and terminology used in the mensuration of rectangles, parallelograms, triangles.
(6) Use of + and − as signs of quality in thermometer readings.

II. The pupil should have considerable skill in the following:

1. Adding, subtracting, multiplying. dividing, and factoring, and checking these processes as applied to whole numbers and to easy fractions, common and decimal.

2. Rhythmic addition, subtraction, and in giving products in number series, such as: Two 8's are 16; how many 16's are four 8's, six 8's, etc.
3. Measuring distances, surfaces, volumes, weight, time, etc.
4. Finding one-digit prime factors.
5. Measuring rectangles, parallelograms, and triangles, and cubical contents of right prisms.
6. Finding the percentage and the percent with small numbers.
7. Finding the simple interest at 6% for times expressed in years.
8. Simplifying common fractions with one-, two-, and three-digit terms.

III. Ideas and values in arithmetical study, advanced or begun in Part II.

1. Heightening interest in arithmetical processes as instruments for skillful measuring.
2. Estimating, judging, and calculating with roughly approximate numbers what results will be.
3. Independence and self-confidence cultivated through the practice of checking, and testing work.
4. Using pencil only when problems are too difficult mentally.
5. Training in analysis by telling how pages of problems should be solved before actually solving them.
6. Usefulness of arithmetic shown through lists of unified problems dealing with school, industrial, and business interests.
7. Adding to the pupils' interest in arithmetic by matters of concern to them, and by strengthening the feeling of power to do arithmetical work.
8. Developing an intelligent interest in the growth and extent of the nation.
9. Developing a sound interest in arithmetic itself.